Applications of Image Processing and Registration in Medical Imaging

Breast CT

Building an Imaging Machine

System Control Modules
- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

ECE 253A Image Processing

Breast CT - Design / Construction

System Modules
Acquisition
Reconstruction
Visualization
Mammogram Simulation
Classification & Extraction
Registration

The Future

Applications of Image Processing and Registration in Medical Imaging

CAD Engineering

X-ray tube & power supply
- 100 kVp @ 4 mA → 0.7 kW generator
- 0.4 mm x 0.4 mm focal spot
- Be window
- water cooled heat exchanger
- high voltage cables
- grounding rod

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control

Applications of Image Processing and Registration in Medical Imaging

System Control Modules

- interface all sub-systems to master control
- position synchronization
- control timing
- control x-ray on
- interlocks
- acquisition control
Applications of Image Processing and Registration in Medical Imaging

X-ray detector
- Solid-state CsI
- TFT detector
  - 1x1: 2048 x 1536
    - 0.194 mm pixels
    - 60 frames per second
  - 2x2: 1024 x 768
    - 0.388 mm pixels
    - 30 frames per second
  - 2x4: 1024 x 384
    - Dual Gain Mode
    - 30 frames per second

Gantry motion
- motor control (angle & motion)
- interface with x-ray
- interface with detector
- support x-ray tube
- support detector
- precise angular control

Kollmorgen Servo Motor
DDR D081M
13.0 ft-lb continuous torque
32.0 ft-lb peak torque

Patient support
- provide comfortable support for patient
- provide close access to chest wall
- support weight of large patients

Data acquisition and analysis
- acquire images through 2π geometry
- incorporate system calibration parameters
- correct images
  - non-linearity
  - pixel drop-out
- perform weighted back-projection algorithm
- convert slices to CT numbers
- provide image orientation
- provide image position

Stationary stand
- Pb shot in foam board
- BB paths are fit to analytical functions that define CT geometry
Applications of Image Processing and Registration in Medical Imaging

Subjective Assessment

The Broccoli Test

Breast CT voxels are 16x Smaller Than Mammography
Applications of Image Processing and Registration in Medical Imaging

**Visualization**
- Slices
- Volume rendering
- Interactivity

**Analysis**
- Segmentation & classification
- Histogram analysis
- Region growing
- Quantification
- Skin
- Fat
- Gland

Breast Tissue Classification

**Joint Probabilities → Tissue Probability Maps**

Joint histograms on maps of discrete slices of multiple images showing, when combined, where these can be located.
Applications of Image Processing and Registration in Medical Imaging

Mammogram Simulation
- synthetic mechanical compression
- finite element analysis (FEA)
- Young's Modulus
- Poisson's ratio
- 3D \rightarrow 2D projection

Breast Compression Models
Primary challenges to simulate mechanical compression of the breast:
+ describe the mechanical properties of breast tissue
+ tissues present
+ proper classification
+ mechanical coefficients for each tissue
+ select the appropriate numerical model
+ describe the contact between the compression paddle and the breast skin
+ mammography unit and the breast skin

Background
Other work

Breast Compression Models
Numerical Compression Methods:
- Evaluated various approaches to compression
  - Forward approach - Finite Element Analysis
  - Inverse approach - registration / transform

Methods
- Finite Element Analysis

Challenges
- Computational time
- Mesh generation
- Mesh complexity

Benefits
- Accurate simulation
- Results may be generalized for rapid computation
- Physical parameters may be extracted from solution

Use techniques of Garboczi et al. to solve the equations in a random linear elastic material, subject to an applied macroscopic strain, using the finite element method.
Simple Compression - Sphere Shell

Sphere shell (Young's Modulus - 2 kPa - Medium 2 kPa)

Sphere shell (Young's Modulus - 90 kPa Medium 2 kPa)

Simple Compression - Sphere Shell

Sphere shell same Young's Modulus

Simple Compression - Sphere Shell

Sphere shell different Young's Modulus

Simple Compression - Breast (segmented)

Breast (segmented) (Young's Modulus - skin 90 kPa, Gland 10 kPa, Fat 1 kPa, Medium 1 kPa)

Simple Compression - Breast (segmented)

Breast segmented Young's Modulus

Applications of Image Processing and Registration in Medical Imaging

Image Registration

• Define a transform $T$ that will map one image onto another image of the same object such that some image quality criterion is maximized.

• Feature Selection:
  • Given $d$ features, find the best subset of size $m$, and
  • "Best" can be defined as
    • minimizing the classification error
    • maximizing discrimination ability of feature set
Applications of Image Processing and Registration in Medical Imaging

Registration Challenges - Strategy
• Transformation Model
• Similarity Measure
• Interpolation Strategy
• Parameter Optimization Strategy

Transformation (T) to map x,y in I₁(x,y) into x',y' in I₂(x,y)

Image Registration

Scheme for Image Registration

Measure
• Entropy
• Mutual Information
• Correlation
• Error

Transformation
• Rigid
• Affine
• Perspective
• Curved

Implementation
• Interpolation
• pdf Estimation
• Optimization
• Acceleration

Similarity Measure
• error
• correlation coefficient
• mutual information
• normalized mutual information

Error estimate:
• compute difference in pixel values between two images
• error = \sum (I₁(x,y) - I₂(x,y))²

Image probability distribution:
• Generate a histogram where the area is the number of possible greyscale values in the image
• Increment histogram cell for each pixel intensity I₁(x,y)
• probability distribution for occurrence of value I in image

Measures of Information
• Shannon's Entropy
  \[ H = \sum p_i \cdot \log \frac{1}{p_i} \]
• weights the information based on the probability that an outcome will occur
• second term shows the amount of information an event provides is inversely proportional to its probability of occurring
Joint image probability distribution:
- Compute the joint histogram \( h(a,b) \) of images.
- Each entry is the number of times an intensity \( a \) in one image corresponds to an intensity \( b \) in the other.
- If the images are perfectly aligned, the histogram is highly focused. As the images mis-align, the dispersion grows.
- Entropy is a measure of histogram dispersion.

Entropy for Image Registration:
- Using joint entropy for registration:
  - Define joint entropy to be:
    \[
    H(A,B) = - \sum_{i,j} p(i,j) \log[p(i,j)]
    \]
  - Images are registered when one is transformed relative to the other to minimize the joint entropy.
  - The dispersion in the joint histogram is thus minimized.

Mutual Information (MI):

1. \( I(A,B) = H(B) - H(B|A) = H(A) - H(A|B) \)
   - Mutual information is the amount that the uncertainty in \( B \) (or \( A \)) is reduced when \( A \) (or \( B \)) is known.
2. \( I(A,B) = H(A) + H(B) - H(A,B) \)
   - Maximizing the mutual info is equivalent to minimizing the joint entropy (last term).
   - Advantage in using mutual info over joint entropy is it includes the individual inputs' entropy.
   - Works better than simply joint entropy in regions of image background (low contrast) where there will be low joint entropy but this is offset by low individual entropies as well so the overall mutual information will be low.

Properties of Mutual Information:
- MI is symmetric: \( I(A,B) = I(B,A) \)
- \( I(A,B) \neq H(A) \) if \( A,B \) are not independent.
- The mutual information can be non-negative:
  - Cannot increase uncertainty in \( A \) by knowing \( B \).
  - If \( A,B \) are independent then \( I(A,B) = 0 \).
  - If \( A,B \) are Gaussian then:
    \[
    I(A,B) = -\frac{1}{2} \log(1 - \rho^2)
    \]

Normalized Mutual Information:
- Two definitions for normalizing Mutual information:
  - Normalized Mutual Information:
    \[
    NMI(A,B) = \frac{I(A,B)}{\sqrt{H(A)H(B)}}
    \]
  - Entropy Correlation Coefficient:
    \[
    ECC(A,B) = 2 - \frac{2}{NMI(A,B)}
    \]
MI Processing Image Registration

Input Images

Pre-processing

Transform Model
- rigid body
- affine
- non-linear
- fluid
- spline

Probability Density Estimation

MI Estimation

Image Transformation

Optimization Scheme

Output Image

Applications of Image Processing and Registration in Medical Imaging

Transformation Model
- rigid body
- affine
- non-linear
- fluid
- spline

Applications of Image Processing and Registration in Medical Imaging

Affine Transforms
- Rigid-body transformations are a subset
- Parallel lines remain parallel
- Operations can be represented by:
  \[ x' = m_{11}x + m_{12}y + m_{13}z + m_{14} \]
  \[ y' = m_{21}x + m_{22}y + m_{23}z + m_{24} \]
  \[ z' = m_{31}x + m_{32}y + m_{33}z + m_{34} \]
- Or as matrices:
  \[ \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \]

Applications of Image Processing and Registration in Medical Imaging

2D Affine Transforms
- Translations by \( t_x \) and \( t_y \)
  \[ x' = x + t_x \]
  \[ y' = y + t_y \]
- Rotation around origin by \( \Theta \) radians
  \[ x' = \cos(\Theta) x - \sin(\Theta) y \]
  \[ y' = \sin(\Theta) x + \cos(\Theta) y \]
- Zoom by \( s_x \) and \( s_y \)
  \[ x' = s_x x \]
  \[ y' = s_y y \]
- Shear
  \[ x' = x + h y \]
  \[ y' = y \]

Applications of Image Processing and Registration in Medical Imaging

Polynomial Basis Functions
\[ x' = a_{11} + a_{12} x + a_{13} x^2 \]
\[ + a_{44} y + a_{45} xy \]
\[ + a_{16} y^2 \]
\[ y' = a_{21} + a_{22} x + a_{23} x^2 \]
\[ + a_{24} y + a_{25} xy \]
\[ + a_{26} y^2 \]

As used by Roger Woods AIR Software

Applications of Image Processing and Registration in Medical Imaging

Cosine Transform Basis Functions

As used by SPM software

11
Local Basis Functions
- More detailed deformations use lots of basis functions with local support.
- Local support means that the basis functions are mostly all zero.
- Faster computations.

Priors enforce "smooth" deformations
- Membrane Energy
  \[-\log(Y) = \frac{1}{2} \int \left( \frac{\partial^2 Y}{\partial x^2} \right)^2 dx\]
- Bending Energy
  \[-\log(Y) = \frac{1}{2} \int \sum_i \left( \frac{\partial^2 Y}{\partial x_i^2} \right)^2 dx\]
- Linear Elastic Energy
  \[-\log(Y) = \frac{1}{2} \int \sum_i \sum_j \left( \frac{\partial^2 Y}{\partial x_i \partial x_j} \right)^2 dx\]

Interpolation Strategy
- Nearest neighbor
- Bi or tri-linear
- Spline

Simple Interpolation
- Nearest neighbor
  - Take the value of the closest voxel.
- Tri-linear
  - Just a weighted average of the neighboring voxels.
  - \( f_5 = f_1 x_2 + f_2 x_1 \)
  - \( f_6 = f_1 x_3 + f_2 x_2 \)
  - \( f_7 = f_1 y_2 + f_2 y_1 \)
B-spline Interpolation

A continuous function is represented by a linear combination of basis functions.

2D B-spline basis functions of degree 0, 1, 2 and 3.

B-splines are piecewise polynomials.

Nearest neighbour and trilinear interpolation are the same as B-spline interpolation with degrees 0 and 1.

Exhaustive search

Alternate between optimizing different groups of parameters.

Steepest descent search

Alternate between optimizing different groups of parameters.

Parameter Optimization Strategy

- exhaustive search
- trial & error
- analytic optimization

Parameter Optimization Strategy - COM

```
for(i = 0 to n)
    for(j = 0 to n)
    {
        sumx1 = sumx1 + pixel_1[j][i] * i
        sumy1 = sumy1 + pixel_1[j][i] * j
        sumx = sumx + pixel_1[j][i]
        sumy = sumy + pixel_1[j][i]
    }
comx = sumx1/sumx <-- may be fractional pixel
comy = sumy1/sumy <-- may be fractional pixel
shftx1 = comx - 128.0
shfty1 = comy - 128.0
...```

Parameter Optimization Strategy
• analytic optimization (COM + moments)

for (i = 0 to n)
for (j = 0 to n)
{
  sumyy = sumyy + pixel_1[j][i] * ((float)i - comx) * ((float)i - comx)
  sumxx = sumxx + pixel_1[j][i] * ((float)j - comy) * ((float)j - comy)
  sumyx = sumyx + pixel_1[j][i] * ((float)i - comx) * ((float)j - comy)
  sumxy = sumxy + pixel_1[j][i] * ((float)i - comx) * ((float)j - comy)
}

sumxx = sumxx/sumx
sumyy = sumyy/sumy
sumxy = sumxy/sumx
sumyx = sumyx/sumy

x0 and y0 are the coordinates of the center of a region R with the area F.
Then the moments M_{ij} are defined by:
M_{ij} = \sum ( (x0 - x)^i (y0 - y)^j ),
where x and y run through all pixels of the region R.
If the moments M_{20}, M_{02} and M_{11} are normalized to the area,
the radii R_a and R_b are calculated as:
R_a = \sqrt{8.0 (M_{20}+M_{02}+ \sqrt{(M_{20}-M_{02})^2+4.0 M_{11}^2})}/2.0
R_b = \sqrt{8.0 (M_{20}+M_{02}- \sqrt{(M_{20}-M_{02})^2+4.0 M_{11}^2})}/2.0
The orientation \Phi is defined by:
\Phi = -0.5 * \arctan2(2.0 * M_{11}, M_{02} - M_{20})

\theta_1 = -0.5 * \arctan2(2*sumxy,(sumxx-sumyy))
Applications of Image Processing and Registration in Medical Imaging

Summary - Image Registration

<table>
<thead>
<tr>
<th>Measure</th>
<th>Transformation</th>
<th>Implementation</th>
</tr>
</thead>
</table>

Applications of Image Processing and Registration in Medical Imaging

Thomas R. Nelson, Ph.D.
Professor of Radiology
UC San Diego
tnelson@ucsd.edu

References