Problem 6.1: Simplify the following expressions involving the impulse and/or shifted impulse and sketch the results:

(a) \( f(t) = (1 + t^2)(\delta(t) - 2\delta(t - 4)) \)
(b) \( g(t) = \cos(2\pi t)(\frac{du(t)}{dt} + \delta(t + 0.5)) \)
(c) \( h(t) = \sin(2\pi t)\delta(0.5 - 2t) \)
(d) \( y(t) = \int_{-\infty}^{\infty} (\tau^2 + 6)\delta(\tau - 2)d\tau \)
(e) \( z(t) = \int_{0}^{\infty} (\tau^2 + 6)\delta(\tau - 2)d\tau \)
(f) \( a(t) = \int_{-\infty}^{t} \delta(\tau + 1)d\tau + \text{rect}(t/6)\delta(t - 2) \)
(g) \( b(t) = \delta(t - 3) * u(t) \)
(h) \( c(t) = \Delta(t/2) * (\delta(t) - \delta(t + 2)) \)

Problem 6.2: Determine the Fourier transform of the following signals – simplify the results as much as you can and sketch the result if it is real valued.

(a) \( f(t) = 5 \cos(5t) + 3 \sin(15t) \)
(b) \( x(t) = \cos^2(6t) \)
(c) \( y(t) = e^{-t}u(t) * \cos(2t) \)
(d) \( z(t) = (1 + \cos(3t))e^{-t}u(t). \)

Problem 6.3: Determine the inverse Fourier transform of the following:

(a) \( F(\omega) = 2\pi[\delta(\omega - 4) + \delta(\omega + 4)] + 8\pi\delta(\omega) \)
(b) \( A(\omega) = 6\pi \cos(5\omega) \)
(c) \( B(\omega) = \sum_{n=-\infty}^{\infty} \frac{2\pi}{2 + \pi} \delta(\omega - 2n) \)
(d) \( C(\omega) = \frac{8}{j\omega} + 4\pi\delta(\omega) \)

Problem 6.4:
(a) Show that the exponential Fourier series of
\[ f(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \]
with period \( T \) is
\[ f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn(2\pi/T)t} \]

(b) Using the result of part (a) as well as the fact that
\[ e^{j\omega_0 t} \leftrightarrow 2\pi \delta(\omega - \omega_0) \]
(see item 17 in Table 7.2 of the textbook) verify the fact that
\[ \sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \sum_{n=-\infty}^{\infty} \frac{2\pi}{T} \delta(\omega - n\frac{2\pi}{T}) \]
(item 24 in Table 7.2 of the textbook).

**Problem 6.5**: Using the result of part (b) of the previous problem, show that
\[ \sum_{n=-\infty}^{\infty} f(t) \delta(t - nT) \leftrightarrow \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\frac{2\pi}{T}) \]
(item 25 in Table 7.2 of the textbook).

**Problem 6.6**: Using the result of the previous problem, and the fact that
\[ \text{sinc}(\pi t/T) \leftrightarrow T \text{rect} \left( \frac{\omega}{2\pi/T} \right) \]
show that
\[ \text{sinc}(\pi t/T) * \sum_{n=-\infty}^{\infty} f(t) \delta(t - nT) = f(t) \]
provided that the bandwidth of \( F(\omega) \) is less than \( \pi/T \) radians/sec. Note that the solution of this problem constitutes an independent proof of the Nyquist criterion as well as the derivation of the reconstruction formula.