Problem 2.1:
Applying the phasor method to the phasor equivalent circuit
we have

\[ I_2 \cdot 1 = I_1 \left( j\omega + \frac{1}{j2\omega} \right), \]
\[ F = (I_1 + I_2) \cdot 1 + I_2 \cdot 1, \]
\[ Y = I_1 \cdot \frac{1}{j2\omega}. \]

Thus the frequency response is

\[ H(\omega) = \frac{Y}{F} = \frac{1}{2 - 4\omega^2 + j2\omega}. \]

Plugging \( \omega = 0 \) into \( H(\omega) \), the DC response is \( H(0) = 0.5 \).

Problem 2.2:
Applying the phasor method to the ODE, we have

\[ (j\omega)^2 Y + j4\omega Y + 4Y = j\omega F. \]

Thus the frequency response is

\[ H(\omega) = \frac{Y}{F} = \frac{j\omega}{4 - \omega^2 + j4\omega}. \]

Problem 2.3:

1. The DC response is \( H(0) = \frac{1}{2} \frac{A}{V} \). So the output is

\[ y(t) = H(0) f(t) = \frac{1}{2} \times 4 = 2A. \]

2. The input phasor is \( F = 2, \omega = 2 \). By the input output relation based on frequency response, we have

\[ Y = H(2) F = \frac{2}{(1 + j2)(2 + j2)} = \frac{-1 - j3}{10} = \frac{1}{\sqrt{10}} e^{j(\tan^{-1}(3) - \pi)} = 0.316 \angle -108.43^\circ. \]

The steady-state output is

\[ y(t) = \frac{1}{\sqrt{10}} \cos(2t + \tan^{-1}(3) - \pi)) = 0.316 \cos(2t - 108.43^\circ). \]
3. Let \( f(t) = f_1(t) + f_2(t) \), where \( f_1(t) = \cos(2t - 10^\circ) \) and \( f_2(t) = 2\sin(4t) \). Then the corresponding phasor is

\[
F_1 = 1 - 10^\circ, \omega_1 = 2 \quad \text{and} \quad F_2 = -j2, \omega_2 = 4.
\]

By the input output relation based on frequency response, we have

\[
Y_1 = H(2)F_1 = \frac{1 - 10^\circ}{(1 + j2)(2 + j2)} = \frac{1}{2\sqrt{10}} e^{j(\tan^{-1}(3) - 10^\circ)} = 0.158\angle -118.43^\circ,
\]

\[
Y_2 = H(4)F_2 = \frac{-j2}{(1 + j4)(2 + j4)} = \frac{-6 + j7}{85} = \frac{1}{\sqrt{85}} e^{j(\pi - \tan^{-1}(1/2))} = 0.108\angle 130.60^\circ.
\]

By superposition, the steady-state output is

\[
y(t) = y_1(t) + y_2(t) = \frac{1}{2\sqrt{10}} \cos(2t + \tan^{-1}(3) - \pi - 10^\circ) + \frac{1}{\sqrt{85}} \cos(4t + 130.60^\circ)
\]

\[
= 0.158 \cos(2t - 118.43^\circ) + 0.108 \cos(4t + 130.60^\circ).
\]

**Problem 2.4:**

From Problem 2, the frequency response is

\[
H(\omega) = \frac{1}{2 - 4\omega^2 + j2\omega}.
\]

Let \( f(t) = f_1(t) + f_2(t) \), where \( f_1(t) = 4 \) and \( f_2(t) = \cos(2t) \). The DC response to \( f_1(t) = 4 \) is

\[
y_1(t) = H(0)f_1(t) = 1/2 \times 4 = 2.
\]

The phasor of \( f_2(t) \) is \( F_2 = 1, \omega_2 = 2 \). By the input output relation based on the frequency response, we have

\[
Y_2 = H(2)F_2 = \frac{1}{2 - 16 + j4} = \frac{1}{2\sqrt{53}} e^{j(\tan^{-1}(1/2) - \pi)} = 0.0687\angle -164.05^\circ.
\]

By superposition, the steady-state output is

\[
y(t) = y_1(t) + y_2(t) = 2 + \frac{1}{2\sqrt{53}} \cos(2t + \tan^{-1}(2/7) - \pi) + 0.0687 \cos(2t - 164.05^\circ).
\]

**Problem 2.5:**

Let \( f(t) = f_1(t) + f_2(t) + f_3(t) \), where \( f_1(t) = 5, f_2(t) = 4e^{j2t} \) and \( f_3(t) = 4e^{-j2t} \). By property 5 in Table 5.1 in the text, the corresponding output is

\[
y_1(t) = H(0)f_1(t) = 5 \times \frac{1}{2} = 2.5,
\]

\[
y_2(t) = H(2)4e^{j2t} = \frac{1 + j2}{2 + j2} 4e^{j2t} = (3 + j)e^{j2t},
\]

\[
y_3(t) = H(-2)4e^{-j2t} = \frac{1 - j2}{2 - j2} 4e^{-j2t} = (3 - j)e^{-j2t}.
\]
By superposition, the steady-state output is
\[ y(t) = 2.5 + (3 + j)e^{j2t} + (3 - j)e^{-j2t} = 2.5 + 2\sqrt{10}\cos(2t + \tan^{-1}\left(\frac{1}{3}\right)). \]

Alternatively, let \( f(t) = f_1(t) + f_2(t) \), where \( f_1(t) = 5 \) and \( f_2(t) = 4e^{j2t} + 4e^{-j2t} = 8\cos(2t) \). The phasor of \( f_2(t) \) is \( F_2 = 8, \omega_2 = 2 \). By input output relation based on frequency response, we have
\[ Y_2 = H(2)F_2 = \frac{1 + j2}{2 + j2} \times 8 = 6 + j2 = 2\sqrt{10}e^{j\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}. \]

By superposition, the steady-state output is
\[ y(t) = y_1(t) + y_2(t) = 2.5 + 2\sqrt{10}\cos(2t + \tan^{-1}\left(\frac{1}{3}\right)). \]

**Problem 2.6:**

Let \( f(t) = f_1(t) + f_2(t) \), where \( f_1(t) = 2e^{-2jt} + 2e^{2jt} \) and
\[ f_2(t) = (2 + 2j)e^{-jt} + (2 - 2j)e^{jt} = 2\sqrt{2}e^{j\pi/4}e^{-jt} + 2\sqrt{2}e^{-j\pi/4}e^{jt} = 4\sqrt{2}\cos(t - 45^\circ). \]

From Problem 12, we know the steady-state output to \( f_1(t) \) is
\[ y_1(t) = \frac{1}{2} \times 2\sqrt{10}\cos(2t + \tan^{-1}\left(\frac{1}{3}\right)) = \sqrt{10}\cos(2t + \tan^{-1}\left(\frac{1}{3}\right)). \]

The phasor of \( f_2(t) \) is \( F_2 = 4\sqrt{2} \angle -45^\circ \). By the input output relation based on frequency response, we have
\[ Y_2 = H(1)F_2 = \frac{1 + j}{2 + j} \times 4\sqrt{2} \angle -45^\circ = \frac{8}{\sqrt{5}}e^{j\left(\tan^{-1}\left(\frac{1}{4}\right) - \frac{\pi}{4}\right)}. \]

By superposition, the steady-state output is
\[ y(t) = y_1(t) + y_2(t) = \sqrt{10}\cos(2t + \tan^{-1}\left(\frac{1}{3}\right)) + \frac{8}{\sqrt{5}}\cos(t + \tan^{-1}\left(\frac{1}{3}\right) - \frac{\pi}{4}) \]
\[ = 3.162\cos(2t + 18.43^\circ) + 3.578\cos(t - 26.57^\circ). \]

**Problem 2.7:**

A LTI system allows amplitude scaling and phase shift. However, it preserves the frequency of the input signal. As a result, we have the following:

1. Yes. Because there are only amplitude scaling and phase shift in the output signal.
2. No. Because there is a new frequency component \( \omega = 0 \) in the output signal.
3. Yes.
4. No. Because the DC response should be real valued.
5. No. Because there is a new frequency component \( \omega = 3 \) in the output signal.
6. Yes.
7. No. Because there is a new frequency component \( \omega = 2\pi \) in the output signal.
8. No. Because \( \sin^2(\pi t) = \frac{1}{2}(1 - \cos(2t)) \). There are two new frequency components \( \omega_1 = 0 \) and \( \omega_2 = 2 \) in the output signal.