Name ________________________________

Your UCSD ID Number ________________________________

Signature ________________________________

INSTRUCTIONS
This exam is closed book and closed notes, with the one exception that you are allowed notes on both sides of one sheet of 8.5 x 11.0 inch paper (not tape or stick-ons allowed). No calculators, laptop computers, or other electronic devices are allowed.

Write your answers in the spaces provided. Show all your work. If you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem. Zero credit will be given for correct answers that lack adequate explanation of how they were obtained. There is a maximum total of 40 points on this exam. Simplify your answers as much as possible and leave answers as fractions, not decimal numbers.

GRADING

1. 20 points ____

2. 20 points ____

TOTAL (40 points) ____
**Problem 1 (20 points)**
The circuit shown below has two resistors, one capacitor, one inductor, a voltage source $v_{in}(t) = 4\sin(3t + (\pi/3))$, and a current source $i_{in}(t) = 2\cos(3t + (\pi/3))$ Find the sinusoidal steady-state output voltage $V_o$ indicated in the diagram. Simplify your answer as much as possible and make sure your answer is purely real-valued. If an expression cannot be simplified by hand, it can be left in a simplified form with square roots, trigonometric functions, etc. If it can be simplified by hand and you don’t, then credit will be lost.

**SOLUTION:** Consider the more general circuit diagram with complex impedances:
Problem 1 (continued)

We have the following equations (using KVL and KCL):

\[ V_o = I_3 Z_c \implies I_3 = \frac{V_o}{Z_c} \]
\[ V_{in} = Z_a I_1 + Z_b (I_1 - I_3) = (Z_a + Z_b) I_1 - Z_b I_3 = (Z_a + Z_b) I_1 - (Z_b/Z_c)V_o \]
\[ \implies I_1 = \frac{V_{in} + (Z_b/Z_c)V_o}{Z_a + Z_b} \]
\[ I_{in} = I_3 - I_2 = (V_o/Z_c) - I_2 = (V_o/Z_c) - I_{in} \]
\[ V_o = -I_2 Z_d - I_1 Z_a \]
\[ = Z_d (I_{in} - (V_o/Z_c)) - Z_a \left( \frac{V_{in} + (Z_b/Z_c)V_o}{Z_a + Z_b} \right) \]

\[ \therefore V_o \left( 1 + \frac{Z_d}{Z_c} + \frac{Z_a Z_b}{Z_c(Z_a + Z_b)} \right) = Z_d I_{in} - \left( \frac{Z_a}{Z_a + Z_b} \right) V_{in} \]

Now use specific values. The current source and voltage source phasors are:

\[ I_{in} = 2e^{j\pi/3} = 1 + j\sqrt{3} \quad \text{and} \quad V_{in} = 4e^{j((\pi/3)-(\pi/2))} = 4e^{-j(\pi/6)} = 2(\sqrt{3} - j). \]

Since \( \omega = 3 \), the impedances are \( Z_a = 1, \ Z_b = 3, \ Z_c = (3/2)j, \ Z_d = -3j. \)

Plugging in gives:

\[ V_o(1 - 2 - (j/2)) = (1 + j\sqrt{3})(-3j) - (1/4)2(\sqrt{3} - j) \]
\[ -V_o(1 + (j/2)) = (5/2)(\sqrt{3} - j) \]
\[ V_o = \frac{5(j - \sqrt{3})}{j + 2} \]
\[ |V_o| = 2\sqrt{5} \]
\[ \angle V_o = -\tan^{-1}(1/\sqrt{3}) - \tan^{-1}(1/2) = -\left( \frac{\pi}{6} + \tan^{-1}(1/2) \right) \]
\[ \therefore v_o(t) = 2\sqrt{5} \cos \left( 3t - \frac{\pi}{6} - \tan^{-1}(1/2) \right) \]
Problem 2 (20 points)

Suppose a linear, time-invariant system has frequency response $H(\omega)$ shown below and let the input signal to the system be $f(t) = \cos(t)s(t)$ where $s(t)$ is the periodic square wave shown below:

Find the output $y(t)$ of the system. (Express your answer as a real-valued function, i.e. with no $j$'s.)
Problem 2 (continued)

SOLUTION: The input is a cosine with alternate periods zeroed out. Thus, \( f(t) \) is periodic with period \( T = 4\pi \), \( \omega_0 = 2\pi/T = 1/2 \), and Fourier series \( f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jnt/2} \). Note that \( H(\omega) = H(-\omega) = 1 - \frac{4}{3}\omega \) if \( \omega \in [0, 3/4] \). The output is

\[
y(t) = \sum_{n=-\infty}^{\infty} F_n H(n\omega_0) e^{jnt/2} = \sum_{n=-1}^{1} F_n H(n/2) e^{jnt/2}
\]

(since \( H(\omega) = 0 \) when \( |\omega| \geq 3/4 \).

\[ F_n = \frac{1}{4\pi} \int_{0}^{2\pi} \cos(t) e^{-jnt/2} dt \]
\[ = \frac{1}{8\pi} \int_{0}^{2\pi} (e^{jt(1-\frac{n}{2})} + e^{-jt(1+\frac{n}{2})}) dt \]
\[ = \frac{1}{j(1-n/2)}e^{jt(1-\frac{n}{2})} \bigg|_{0}^{2\pi} - \frac{1}{j(1+n/2)}e^{-jt(1+\frac{n}{2})} \bigg|_{0}^{2\pi} \]
\[ = \frac{1}{j(1-n/2)}(e^{2\pi(1-\frac{n}{2})} - 1) - \frac{1}{j(1+n/2)}(e^{-2\pi(1+\frac{n}{2})} - 1) \]
\[ = \begin{cases} 0 & \text{if } n = 0 \\ \frac{8j}{3} & \text{if } n = 1 \\ -\frac{8j}{3} & \text{if } n = -1 \end{cases} \]

\[ y(t) = F_1 H(1/2) e^{jt/2} + F_{-1} H(-1/2) e^{-jt/2} \]
\[ = \frac{8j}{3} \cdot \frac{1}{3} e^{jt/2} + \frac{-8j}{3} \cdot \frac{1}{3} e^{-jt/2} \]
\[ = \frac{j}{9\pi} (e^{jt/2} - e^{-jt/2}) \]
\[ = -\frac{2}{9\pi} \cdot \sin(t/2) \]