SESSION TP4

QUANTIZATION II

Asymptotics of Quantizers Revisited

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In this paper we reanalyze the results by Gish and Pierce (1968) on asymptotically efficient quantizing. In particular, asymptotic bounds are derived on the difference between the entropy of the uniform quantizer and that of the optimal quantizer when the mean square error becomes small. Hereby, no assumptions are made at all on the density of the random variable being quantized, and use is made of some classical results by Rényi (1959) and Csisár (1973). Also, following the work by Ziv (1985), non-asymptotic and distribution-free bounds on the difference between the entropy of the uniform quantizer and that of the optimal quantizer are derived if both have the same distortion. Finally, non-uniform quantizers are considered. For the latter case the asymptotic relation is investigated between the entropy of the quantizer and the entropy of the random variable being quantized, with no assumption at all on the density.

Low Dimension/Moderate Bit Rate Vector Quantizers for the Laplace Source

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In this paper we present an *unrestricted vector quantizer* for the independent Laplace source $x(x \in \mathbb{R}^k)$, a source model useful for speech and image coding. The VQ is based upon Helmert's transformation

$$g = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |x_i| \; ; \quad u_j = \frac{1}{\sqrt{j(j+1)}} \left[\sum_{i=1}^{j} |x_i| - j\,|x_{j+1}| \right], \quad j = 1, 2, \ldots, k-1$$

and consists of a scalar quantizer for g and a lattice-based uniform VQ on a dimension $k-1$ simplex, $\alpha_{k-1}(\hat{g})$, for the u_i. The VQ is unrestricted in that the resolution of the lattice VQ for the u_i varies with the result of the g quantization.

Asymptotic performance and design results are reviewed. Fischer's pyramid vector quantizer is shown to be a special case of our VQ; low-dimension results for the PVQ are provided. Implementation details of the VQ using various lattices are presented. Due to the geometric complexity of an exact analysis, an approximate design algorithm for finite bitrate is developed. Example designs with Monte Carlo performance simulations are presented for low dimension and moderate bitrates (2 through 4 bits per dimension) to demonstrate the utility of this approach.

Source/Channel Coding for Vector Quantizers by Index Assignment Permutations

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An *index assignment function* for a vector quantizer in the presence of channel noise is a permutation of the quantizer codevector indices. Each output index from a vector quantizer is mapped by an index assignment function to a new index which is subsequently used as the input to a block channel coder. For a given vector quantizer (source coder) and a given error control block channel code, the choice of an index assignment function can have a profound effect on the average distortion achieved.

This paper shows that existing procedures for the design of index assignment functions in the zero redundancy case can be generalized to include the use of block channel coding of the permuted indices. An effective design method is introduced for constructing index assignment functions for arbitrary block

error control codes with the objective of minimizing average distortion. Extensive numerical results for first-order Gauss-Markov and speech sources and for systematic linear codes with varying code rates demonstrate substantial performance gain at various channel error rates. Often, a gap of several dB of signal-to-noise ratio can exist between a poor and a good index assignment function. One interesting consequence is that a good index assignment for a quantizer with no channel with no coder can often be superior in performance to a poor index assignment for the same quantizer using redundancy coding.

Optimal Quantization over a Finite-State Noisy Channel

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The problem of quantizer design when the quantizer output is transmitted over a finite-state noisy channel is considered. Such a channel is a good model for fading channels, and the channel state information may be available to the receiver through signal-to-noise ratio measurements. Necessary optimality conditions are derived for the encoder when the decoder is fixed and given, and vice versa. In the absence of channel state information at the decoder, the channel is simply equivalent to a discrete memoryless channel, and previous results and methods are applicable. When side information is available, it can be used by the decoder along with channel outputs to better reproduce the source information. In either case, the optimality conditions lead to an iterative design algorithm. Numerical results are presented showing the performance of the system for various sources, channel models, and transmission rates. It is shown that side information may lead to noticeable improvements in performance.

Reduced Complexity Entropy-Pruned Tree-Structured Vector Quantization

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Vector quantization is a data compression technique in which a vector of source symbols is represented by a reproduction vector drawn from a finite codebook of such vectors. The index of the chosen vector is all that need be communicated. Since the probability distribution across codebook indices is not uniform, additional compression can be achieved by application of variable rate noiseless entropy coding to the indices.

Performance gains can be achieved if the vector quantizer and entropy code are designed jointly instead of sequentially. Entropy-pruned tree-structured vector quantization is a computationally attractive jointly designed system in which a large tree-structured vector quantizer is pruned so as to produce the smallest distortion among all pruned trees of the same or lesser entropy. In this paper, I show how binary arithmetic coding applied at each node of the tree search produces a system that is capable of progressive transmission and is also well suited to buffer-instrumented communication systems. In fact, the multiplication free Q-coder arithmetic code combined with a piecewise linear distortion function in the vector quantizer leads to a multiplication free encoder with good rate-distortion performance and significant flexibility. Simulations are performed for image coding experiments.

Optimal Scalar and Vector Predictive Quantizer Design

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Differential Pulse Code Modulation (DPCM) systems are important systems for data compression of speech, images and other signals. An optimal DPCM system based on optimization theory and Lloyd-Max quantizers is developed. Its performance is compared to a matched DPCM system for first order Gauss-Markov sources with a mean square error performance criterion. The performance of the optimal DPCM system is found to be closer to the distortion rate bound at high bit rates than the conventional