Universal Adaptive Vector Quantization using Codebook Quantization with Application to Image Compression

Kenneth Zeger and Anurag Bist

University of Hawaii, Dept. of Elect. Eng.
Holmes Hall 483, Honolulu, HI 96822

Abstract

A high resolution analysis is presented for a universal vector quantization scheme based on periodic codebook transmissions. The scheme assumes a slowly changing nonstationary source such as an image and periodically transmits new updated codebooks as side information to the receiver. The side information is transmitted via a large universal codebook which itself acts as a quantizer for the updated codebooks to be transmitted. This scheme generalizes the more simple technique of adapting a codebook by transmitting its vector components one at a time using a fixed uniform scalar quantizer. These schemes are compared both theoretically and experimentally and an optimum tradeoff between quantization resolution and side information is determined.

1 Introduction

Vector Quantization (VQ) plays a critical role as an important building block of many lossy data compression systems and is generally designed based on the long term statistical behavior of a source. In many situations, however, (e.g. image coding) nonstationary sources are encountered where real-time adaptation is desirable. An approach to providing this need is for a quantizer to be both *adaptive* and *universal* in nature. An adaptive quantizer is one in which changing source statistics induce changes in the quantization procedure or parameters, and a universal quantizer is one which is a priori able to successfully encode a large class of distinct sources. These two notions are very closely related and are encountered in lossless source coding, such as with Ziv-Lempel coding and Gallagher's adaptive Huffman coding. However, for lossy source coding, there is a significant gap in this area.

Ziv [1] has shown that, under some alphabet assumptions, there exist universal algorithms for the class of all stationary sources that asymptotically do as well for each source as an optimum source code designed for that source. Neuhoff et al [2] develop a unified theory for universal source coding and Matsuyama and Gray [3] have applied the idea of universal coding to tree encoding of speech. Recently, Chou [4] designed weighted universal codes for image coding by using a finite collection of predesigned VQ codebooks. A short binary index is occasionally transmitted to specify to the decoder which codebook is being used. This scheme is limited in the sense that the number of different codebooks that can be used is rather small due to memory and complexity constraints.

Nasrabadi [5] used the notion of a "super-codebook", a large ordered codebook available at both the encoder and decoder, from which the first N codevectors serve as an operational vector quantizer in coding an image. The operational codebook can be adapted by trickling vectors in the super-codebook to the top and allowing others to fall down to lower positions. Their reordering method is heuristic in nature, and depends on the statistics of the previously used codevector indices. One potential difficulty with this scheme is that being finite state in nature, it cannot easily recover from channel errors. An earlier method for adaptive quantization was proposed by Gersho and Yano [6] that adaptively replenishes codevectors as the source statistics change, attempting to keep the partial distortions constant at each stage. In [7] subsets of a universal codebook are used as reduced size codebooks, which are then used to encode the source training set. Also, in [7], adaptive quantization is performed by occasionally transmitting (as overhead) new codebooks to a receiver.

In the present paper, we describe and mathematically analyze a universal quantization technique based upon the occasional transmission of new codebooks (see [9] pg. 620). The main idea is that periodically, as the source statistics change, a new *target* VQ codebook, $\mathcal{C}_t$, of size N is designed for the source and then matched in a nearest neighbor manner to the N closest vectors in a large *universal* codebook, $\mathcal{C}_u$, of size M. The N matched vectors from $\mathcal{C}_u$ constitute the *operational* codebook, $\mathcal{C}_o$, which is used for coding by both the encoder and receiver as an approximation to $\mathcal{C}_t$. The operational codebook can be conveyed to the receiver by transmitting side information specifying some N-vector subset of $\mathcal{C}_u$. In this manner, the vector quantizer is itself being vector quantized for the purposes of transmitting its codebook. This technique generalizes and improves upon those described in [4] and [5].

We employ high resolution quantization theory to analyze the performance of this universal coding system. Specifically, the optimal tradeoff between overhead bits used for transmitting new codebooks and the encoding bits sent as codevector indices is determined for various universal codebook designs, and is compared to experimental results. Two methods of codebook transmission are considered: 1) uniform scalar quantizing the codevector components, and 2) vector quantizing the codevectors themselves.

Let $\mathbf{X}$ be a k-dimensional input random vector with pdf $f(\mathbf{x})$, to be quantized by an operational codebook $\mathcal{C}_o$ of size N, which is some subset of a universal codebook $\mathcal{C}_u$ of size M. Define the *codebook ratio* to be the quantity $\beta = M/N$, and assume a new operational codebook is transmitted every α input vectors. Let r denote the system's overall transmission rate in units of bits per scalar sample.

2 Scalar Quantized Codebook

Suppose that the covectors of $\mathcal{C}_o$ are formed by scalar quantizing with b bits each component of every k-dimensional codevector in $\mathcal{C}_t$ (see Fig. 1). Then, equating two expressions for the total number of bits transmitted between codebook updates gives

$$\alpha r k = \alpha \log_2 N + kbN \qquad (1)$$

The research was supported in part, by Hewlett-Packard Co., and the National Science Foundation under Grants No. NCR-90-09766 and NCR-91-57770.

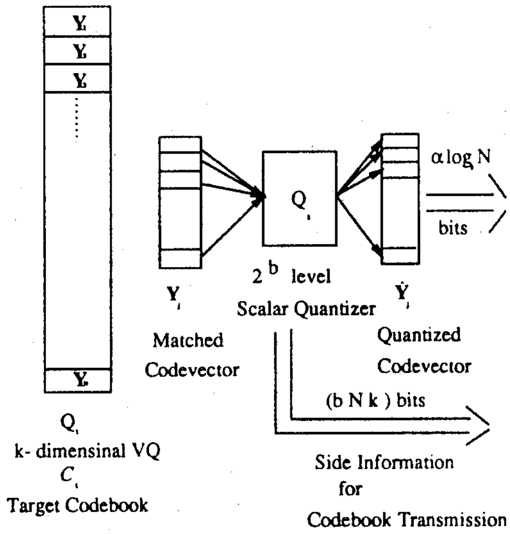

Figure 1: Scalar Quantization of Codebook

The term kbN is the total number of bits used for scalar quantizing $\mathcal{C}_t$ and $\alpha \log_2 N$ is the number of bits transmitted as codevector indices for encoding the source. Solving this equation for b yields

$$b = \frac{\alpha(rk - \log_2 N)}{Nk} \tag{2}$$

Using high resolution quantization theory, we analyze the tradeoff between the number of bits b that are dedicated to quantizing each scalar component of the codebook and the number of bits $\log_2 N$ transmitted to specify each codevector index once encoding begins.

Denote the i^{th} codevector and partition cell of $\mathcal{C}_t$ respectively by $\mathbf{Y_i}$ and R_i and the corresponding quantized approximation codevector in $\mathcal{C}_o$ and its cell by $\hat{\mathbf{Y}}_\mathbf{i}$ and $\hat{R}_i$. For any codebook $\mathcal{C}$ let $D(\mathcal{C})$ denote its mean-square distortion. Then we have:

$$\begin{aligned} D(\mathcal{C}_o) = & \ \tfrac{1}{k}\textstyle\sum_{i=1}^N \hat{P}_i E(||\mathbf{X} - \hat{\mathbf{Y}}_\mathbf{i}||^2 | \mathbf{X} \in \hat{R}_i) \\ \approx & \ \tfrac{1}{k}\textstyle\sum_{i=1}^N P_i E(||\mathbf{X} - \hat{\mathbf{Y}}_\mathbf{i}||^2 | \mathbf{X} \in R_i) \\ = & \ D(\mathcal{C}_t) + \tfrac{1}{k}\textstyle\sum_{i=1}^N P_i ||\mathbf{Y_i} - \hat{\mathbf{Y}}_\mathbf{i}||^2 \end{aligned} \tag{3}$$

where $P_i = Prob[\mathbf{X} \in R_i]$ and $\hat{P}_i = Prob[\mathbf{X} \in \hat{R}_i]$. Thus, the overall quantizer distortion can be approximately decomposed into the distortion of the target quantizer and the distortion incurred in quantizing the codevectors .

Under the assumption of high resolution quantization (N large) [10] we can approximate

$$D(\mathcal{C}_t) = C(k,2) N^{\frac{-2}{k}} ||f(\mathbf{x})||_{k/(k+2)} \tag{4}$$

where the functional $||.||_p$ is given by

$$||f(x)||_p = \left(\int f(x)^p dx \right)^{\frac{1}{p}} \tag{5}$$

and $C(k,2)$ is the coefficient of quantization, which is known to bounded by [10]

$$\frac{1}{k+2} V_k(2)^{-2/k} < C(k,2) < \frac{\Gamma\left(1 + \frac{2}{k}\right)}{k} V_k(2)^{-2/k}$$

where $V_k(2)$ is the volume of a unit sphere in $\mathcal{R}^k$.

The second term of (3) gives the expected distortion in scalar quantizing each component of the codevector. Assuming a uniform scalar quantizer with 2^b output levels over a support region $[-V, V]$, we get

$$\frac{1}{k} \sum_{i=1}^{N} P_i ||\mathbf{Y_i} - \hat{\mathbf{Y}}_\mathbf{i}||^2 = \frac{V^2}{3} 2^{-2b} \tag{6}$$

Substituting (5) and (6) into (3) we have

$$D \approx C(k,2) N^{\frac{-2}{k}} ||f(\mathbf{x})||_{\frac{k}{k+2}} + \frac{V^2}{3} 2^{-2b} \tag{7}$$

Combining equations (2), (3), (4), and (7) gives the overall distortion as a function of N:

$$D(\mathcal{C}_o) = C(k,2) ||f(\mathbf{x})||_{k/(k+2)} N^{-2/k} + \frac{V^2}{3} 2^{\frac{-2r\alpha}{N}} N^{\frac{2\alpha}{Nk}} \tag{8}$$

Large values of N correspond to allocating more of the transmitted bits to codevector indices (i.e. larger codebooks), whereas smaller values of N imply more bits are used as overhead to accurately transmit the operational codebooks. To minimize the overall distortion, we set the derivative of $D(\mathcal{C}_o)$ with respect to N equal to zero and numerically compute the roots of the resulting equation. Section 4 gives results for this scalar quantization of the target codebook designed for a Gaussian iid source.

3 "VQing" the Codebook

In the previous section we discussed the case where we scalar quantized the components of the codebook vectors and transmitted the quantizer output bits to the decoder. We next extend the analysis of the previous section by assuming the target codebook $\mathcal{C}_t$ is transmitted to the receiver by vector quantizing its codevectors to form the operational codebook $\mathcal{C}_o$. The size M codebook $\mathcal{C}_u$ is available to both the encoder and decoder. As in the scalar case, the target codebook is designed in real-time based on the current (but unknown a priori) source statistics. For each codevector in $\mathcal{C}_t$ the nearest codevector in $\mathcal{C}_u$ is determined and put into $\mathcal{C}_o$. If a codevector in $\mathcal{C}_u$ is chosen twice it is thrown out and instead the next nearest unused codevector in $\mathcal{C}_u$ is put into $\mathcal{C}_o$.

Note that the target codebook $\mathcal{C}_t$ is treated as a *vector source* that is itself quantized by $\mathcal{C}_u$. The universal quantizer, however, is mismatched to the statistics of the codebook source $\mathcal{C}_t$ since the input source $\mathbf{X}$ is assumed unknown ahead of time. On the other hand, the quantizer $\mathcal{C}_t$ is assumed to be matched to the statistics of the source $\mathbf{X}$, since it is designed "on the fly" by applying the generalized Lloyd algorithm to recently observed training set vectors. The overall system is shown in Fig 2.

As M increases the operational codebook more closely approximates the target codebook, but more of the available bits must be dedicated to transmitting side-information. We determine the optimum tradeoff between M and N, for a fixed overall rate r (bits/sample), analogous to the previous section. We treat $\mathcal{C}_o$ as a "source" to $\mathcal{C}_u$ and assume its pdf equals the k-dimensional point density function given by asymptotic theory as

$$\lambda_o(\mathbf{x}) = \frac{f(\mathbf{x})^{\frac{k}{k+2}}}{\int_{\mathcal{R}^k} f(\mathbf{x})^{\frac{k}{k+2}} d\mathbf{x}} \tag{9}$$

Following similar steps as in the previous section we get

$$\begin{aligned} D(\mathcal{C}_o) &= D(\mathcal{C}_t) + \frac{1}{k} \sum_{i=1}^{N} P_i ||\mathbf{Y_i} - \hat{\mathbf{Y}}_\mathbf{i}||^2 \\ &\approx C(k,2) ||f(\mathbf{x})||_{k/(k+2)} N^{-2/k} \\ &+ C(k,2) \left(\int_{\mathcal{R}^k} \frac{\lambda_o(\mathbf{x})}{[\lambda_u(\mathbf{x})]^{\frac{2}{k}}} d\mathbf{x} \right) M^{-2/k} \end{aligned} \tag{10}$$

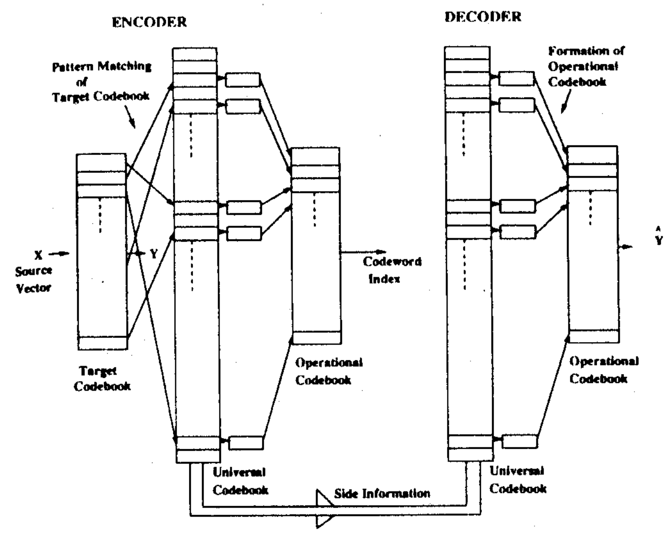

Figure 2: Vector Quantization of Codebook

where in (10) $\lambda_u(\mathbf{x})$ is the point density function of the universal codebook which is a size M VQ codebook designed for a source with pdf $\lambda_o(\mathbf{x})$. The first term in (10) corresponds to the distortion incurred in quantizing a source by a size N VQ codebook of dimension k, and the second term is the distortion for quantizing the target codebook with the universal quantizer.

As before, a block of α k-dimensional source vectors is quantized by $\mathcal{C}_o$ for each new operational codebook transmitted. The number of bits of side-information needed to specify each $\mathcal{C}_o$ is the same as the number of bits required to specify an arbitrary subset of size N from a larger set of size M, namely $\log_2 \binom{M}{N}$. Some implementation complexity is required, but it should be noted that there are also less complex schemes for sending the side-information which respectively use M and $N \log_2 M$ bits [9]. The total number of bits transmitted for every block of α source vectors is

$$\alpha r k = \alpha \log_2 N + \log_2 \binom{M}{N} \qquad (11)$$

where the first term on the right hand side is the number of bits used to transmit the codevector indices during the source quantization and the second term is the side-information. For a given α and N this equation can iteratively be solved for M. Now, the values of $D(\mathcal{C}_o)$ in (10) can be determined using (11).

For $\mathcal{C}_o$ designed for a particular distribution and $\mathcal{C}_u$ designed for a different distribution, this function can be numerically evaluated for different values of N at a fixed rate. In section 4 the minimum distortion bit tradeoff is evaluated, both theoretically and experimentally, for a case where $\mathcal{C}_o$ is Gaussian and $\mathcal{C}_u$ is Laplacian. It is shown that in practical situations, M can become prohibitively large as N decreases (fixed rate). In such cases it is better to have M as large as possible.

4 Experimental Results

Simulations were conducted for various cases for the scalar quantization of the codebook to examine the bit tradeoff. Fig. 3 shows the results for an i.i.d. Gaussian source for rate 2.0. The dimension of $\mathcal{C}_t$ is fixed to be $k = 4$ and b, the number of bits used for scalar quantizing the codebook components, is varied, keeping the overall rate at 2.0. It is seen that the overall SNR first increases and then decreases. The place where the SNR peaks gives the optimum tradeoff of bits. Fig. 4 shows the variation of SNR for the universal quantization scheme where $\mathcal{C}_o$ is Gaussian and $\mathcal{C}_u$ is Laplacian, each with $k = 4$ and $r = 2$ bits per sample. As universal codebook size $M$is varied (for a fixed rate r), we observe a similar tradeoff in the side information as in the scalar quantization of the codebook. Very quickly M becomes prohibitively large as we decrease N for a fixed overall rate. For practical purposes, in this case it would be good to choose as large an M as possible. The differences in the experimental and the theoretical curves are attributed to the approximations made in deriving the asymptotic formulas.

Table 1 gives the results for the universal scheme in coding images. Three test images were taken and a $\mathcal{C}_u$ of size 1792 was designed by running a GLA based on a training set, which was different from the test images. The size of $\mathcal{C}_t$ was 256 in all the cases and we consider encoding at a rate of $r = 0.5$ bits per pixel. The table gives comparative SNRs for encoding of the test images by the universal scheme, ordinary VQ, and the case when they are encoded on a codebook which is designed based on a different image (corresponding to "Mismatch" in the table). Finally, Fig. 5 shows the improvement in performance at the expense of side information.

This shows the effect of increase in the codebook ratio, $\beta = \frac{M}{N}$, for encoding of the image Tiffany. As the codebook ratio increases, the performance also increases, as expected. Though the overall rate increases slightly as we increase the codebook ratio, still the curve is a good indicator of the performance improvement at the expense of side information.

Images	Mismatch			VQ	U-VQ
	Lena	Tiffany	Baboon		
Lena	32.20	20.45	29.70	30.29	31.12
Tiffany	27.60	33.79	22.95	24.63	31.02
Baboon	23.04	19.07	24.50	22.95	23.52

Table 1 : Comparative SNRs of Universal Codebook with Mismatched Codebooks and normal VQ codebooks at 0.5 bpp.

5 Conclusions

We have generalized the concept of codebook quantization for adaptive and universal vector quantization. Using high resolution quantization theory, we have derived explicit formulas for distortion in two different cases. One in which we do a uniform scalar quantization of the initial codebook and the other in which we vector quantize the vector codebook. In both the cases there exists a tradeoff in the number of bits dedicated as side information and the number of bits used in specifying the codebook vectors once the codebook starts getting used. Asymptotic analysis gives a theoretical justification of this tradeoff and this is confirmed by experiments.

References

[1] J. Ziv, "Coding of Sources with unknown statistics - Part 2: Distortion relative to a fidelity criterion," *IEEE Transactions on Information Theory*, Vol.IT-18, pp.389-394, May 1972.

[2] D. L. Neuhoff, R. M. Gray and L. D. Davisson, "Fixed Rate Universal Block Source Coding with a Fidelity Criterion," *IEEE Transactions on Information Theory*, Vol.IT-21, pp.511-523, Sept. 1975.

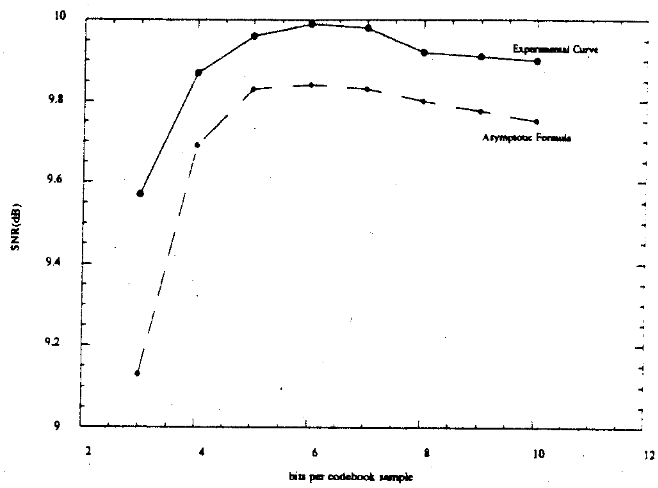

Figure 3: Scalar Quantization of C_t : Gaussian Source, k = 4, rate = 2.0

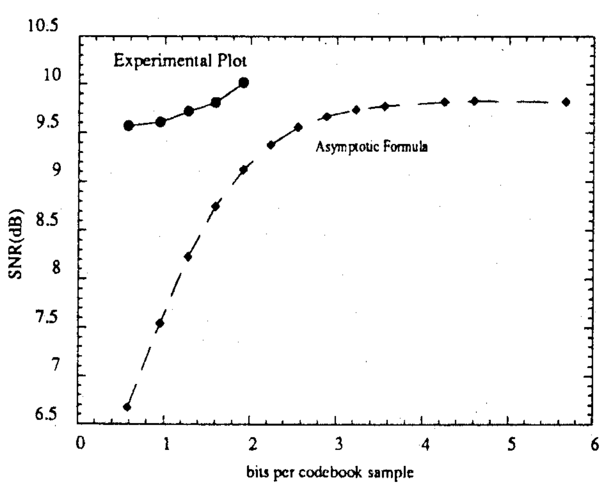

Figure 4: Vector Quantization of C_t : C_o = Gaussian , C_u = Laplacian, k = 4 , rate = 2.0

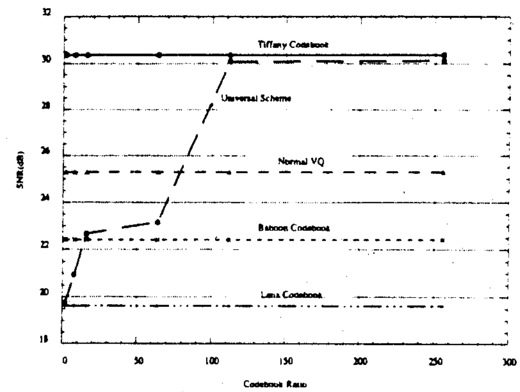

Figure 5: Encoding of Tiffany : rate = 0.25 bpp

[3] Y. Matsuyama and R. M. Gray, "Universal Tree Encoding of Speech," *IEEE Transaction on Information Theory*, Vol.IT-27, pp.31-40, Jan. 1981.

[4] P. A. Chou, "Code Clustering for Weighted Universal VQ and Other Applications," *IEEE International Symposium on Information Theory* (ISIT), Budapest, Hungary, June 1991.

[5] N. M. Nasrabadi and Y. Feng, "A Dynamic Finite-State Vector Quantization Scheme," *Proceedings IEEE International Conference on Acoustics Speech and Signal Processing* (ICASSP), Albuquerque, New Mexico, pp 2261-64, May 1990.

[6] A. Gersho and M. Yano, "Adaptive Vector Quantization by Progressive Codevector Replacement," *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP), May 1985.

[7] S. Panchathan and M. Goldberg, "Algorithms and Architecture for Image Adaptive Vector Quantization," *SPIE, Visual Communications and Image Processing*, Vol. 1001, 1988.

[8] Y. Linde, A. Buzo, R. M. Gray, "An Algorithm for Vector Quantizer design," *IEEE Transactions on Communications*, Vol.COM-28, pp.84-95, Jan. 1980.

[9] A. Gersho and R. M. Gray, "Vector Quantization and Signal Compression," Kluwer Academic Publishers, Boston, 1992.

[10] P.Zador, "Asymptotic quantization error of continuous signals and quantization dimension," *IEEE Transactions on Information Theory*, Vol.IT-28, pp.139-149, March 1982 .

[11] T.D.Lookabaugh and R.M.Gray, "High Resolution Quantization Theory and the Vector Quantizer Advantage," *IEEE Transactions on Inf. Theory*, Vol.IT-35, No.5, Sept. 1989 .

[12] A.Gersho, "Asymptotically optimal block quantization," *IEEE Transactions on Information Theory*, Vol.IT-25, pp.373-380, July 1979 .