On the Performance of Affine Index Assignments for Redundancy Free Source-Channel Coding *

András Méhes Kenneth Zeger

Coordinated Science Laboratory, University of Illinois,
1308 W. Main, Urbana, IL 61801
(email: zeger@uiuc.edu)

An *affine index assignment* is an index assignment of the form $\Pi(i) = iG + d$, where G is called the *generator matrix* (binary) and d is the binary *translation vector*, and the operations are performed over $GF(2)$. An affine assignment with $d = 0$ is called *linear*. Many popular redundancy free codes are linear or affine, including the Natural Binary Code (NBC), the Folded Binary Code (FBC), the Gray Code (GC), and the Two's Complement Code (TCC).

Theorem 1 *The channel distortion of a uniform 2^n level scalar quantizer with step-size Δ, which uses an affine index assignment with generator matrix G to transmit across a binary symmetric channel with crossover probability q, is given by*

$$D_C = \frac{\Delta^2}{4}\sum_{k=0}^{n-1}\sum_{l=0}^{n-1}\left(1 - 2(1-2q)^{w(\vec{f}_{n-k})} + (1-2q)^{w(\vec{f}_{n-k}\oplus\vec{f}_{n-l})}\right)2^{k+l}\hat{P}_{e_k\oplus e_l},$$

where $w(\cdot)$ denotes Hamming weight, $\vec{f}_k = [f_{1,k},\ldots,f_{n,k}]^T$ is the k^{th} column of G^{-1}, $\hat{P}_l$ is the l^{th} component of the Hadamard transform of the induced discrete distribution on the encoder cells, e_k is a binary unit vector with its only nonzero entry in the k^{th} position, and $\oplus$ indicates modulo 2 addition.

Using Theorem 1 we compare the NBC and the FBC for any source distribution.

Corollary 1 *Given a uniform scalar quantizer that transmits across a binary symmetric channel with crossover probability $q < 1/2$ the channel distortion of the Folded Binary Code is larger than that of the Natural Binary Code if and only if,*

$$\mathrm{E}\left[\left(Q(X) - \frac{a+b}{2}\right)^2\right] > \frac{(b-a)^2}{16}.$$

For symmetric distributions, $\mathrm{E}[Q(X)] = \frac{a+b}{2}$, and then we have a threshold decision on the quantizer output variance.

*The research was supported in part by the National Science Foundation.