Structured Spherical Codes for Gaussian Quantization

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Abstract — **We construct a vector quantizer for the Gaussian source which outperforms known existing quantizers for rates above 2 bits per sample, and of equivalent computational complexity. In particular it outperforms "trellis-based scalar-vector quantization" and "trellis coded quantization". The quantizer is a gain-shape quantizer, whose shape codebook consists of a "wrapped spherical code" in 25 dimensions, constructed from the Leech lattice in 24 dimensions.**

I. Introduction

A major goal in source coding theory is to design a quantizer that has both low implementation complexity and performance close to the rate-distortion function of the source. Our proposed quantizer is optimized with respect to a memoryless Gaussian source and has operating complexity that grows linearly with the rate. Applications include quantizing the prediction error signal in a DPCM (differential pulse code modulation) coder for moving pictures, discrete Fourier transform coefficients, and holographic data. Furthermore, a known filtering technique tends to make any memoryless source appear Gaussian, which makes the system insensitive to errors in modeling the input.

II. A Wrapped Spherical Vector Quantizer

Each source vector X is decomposed into *gain* $g = \|X\|$ and *shape* $S = X/g$ components, g and S are quantized separately to $\hat{g}$ and $\hat{S}$, respectively, and the VQ output is $\hat{g}\hat{S}$. For the shape-gain quantizer presented here, $\hat{g}$ depends only on g and $\hat{S}$ depends only on S.

The gain codebook is optimized by the Lloyd-Max algorithm. No training vectors are needed for this since the pdf is known exactly [1]. Furthermore, since $f_g(r)$ is a log-concave function, the Lloyd-Max algorithm converges to the *globally* optimum gain codebook.

The shape codebook is generated by the recently introduced wrapped spherical code as described in [1]. This construction effectively "wraps" any lattice in dimension $k-1$ onto the k-dimensional sphere, with little distortion in the lattice structure.

III. Performance Analysis and Simulations

Omitting algebraic details, the MSE per dimension of the quantizer can be decomposed into shape and gain components:

$$D = \frac{1}{k}E[\|X - \hat{g}\hat{S}\|^2] = D_g + D_s$$

where

$$D_g = \frac{1}{k}E[\|X - \hat{g}S\|^2] = \frac{1}{k}E[(g-\hat{g})^2],$$

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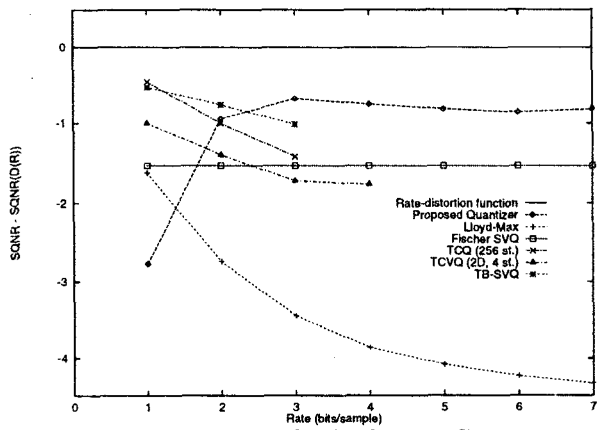

Figure 1: Comparison of VQs for the Gaussian source.

and where

$$D_s = \frac{1}{k}E[\|\hat{g}S - \hat{g}\hat{S}\|^2] \approx \frac{1}{k}E[g^2]E[\|S - \hat{S}\|^2].$$

Sakrison accomplished a similar decomposition for his quantizer [2], but his quantizer had only one gain output value. This convenient decomposition of D into D_s and D_g allows us to optimize the overall quantizer by separately optimizing the shape and gain components.

Using a scalar gain codebook and a 25-dimensional shape codebook constructed from the 24-dimensional Leech lattice, the performance of the quantizer was evaluated with computer-generated i.i.d. Gaussian random samples. The average distortion was computed for 500,000 Gaussian samples, i.e., 20,000 25-dimensional vectors. As indicated in Figure 1 the proposed quantizer performs within one dB of the distortion-rate function for rates in the range of two to seven. For this range, it performs better than some of the best quantizers in the literature.

IV. Conclusions

The wrapped spherical vector quantizer for the memoryless Gaussian source achieves excellent distortion performance, in some cases better than any other published results. This is especially true at high rates. The operating complexity of the quantizer grows linearly with the rate.

References

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