Unachievability of Network Coding Capacity

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Abstract— The coding capacity of a network is the supremum of ratios k/n, for which there exists a fractional (k, n) coding solution, where k is the source message dimension and n is the maximum edge dimension. The coding capacity is referred to as routing capacity in the case when only routing is allowed. A network is said to achieve its capacity if there is some fractional (k, n) solution for which k/n equals the capacity. The routing capacity is known to be achievable for arbitrary networks. We give an example of a network whose coding capacity is not achievable. As part of the proof, we construct two networks, one of which is solvable if and only if the alphabet size is odd, and the other of which is solvable if and only if the alphabet size is a power of two. No linearity assumptions are made.

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