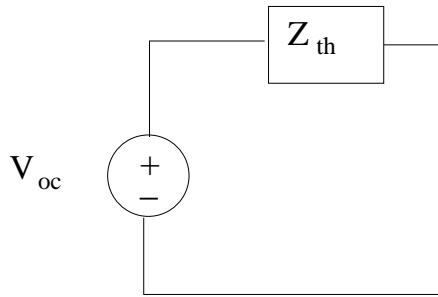
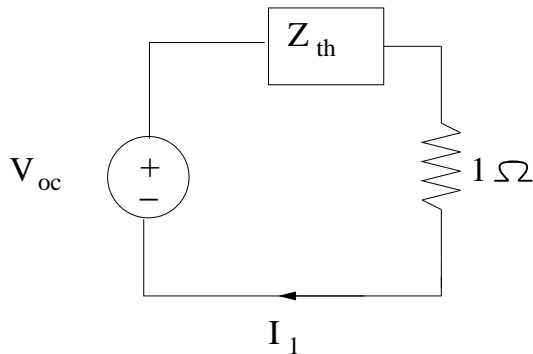


Solutions for Homework 5, problems 6-8

Problem 6: Network A can be represented by its Thevenin equivalent circuit, which looks like this:



When we hook up the 1Ω resistor, the current that flows is $I_1 = 5\angle -45^\circ$.



Likewise when we hook up the Thevenin equivalent to the $1F$ capacitor, the current that flows is $I_2 = 10\angle -45^\circ$.

Two KVL equations can now be written as follows, for these two circuits:

$$V_{oc} = (1 + Z_{th}) \times 5\angle -45^\circ$$

$$V_{oc} = (Z_{th} - j) \times 10\angle -45^\circ$$

Now we have two equations with two unknowns, V_{oc} and Z_{th} . We can equate the right hand sides of the two equations to eliminate V_{oc} and find Z_{th} .

$$(1 + Z_{th}) \times 5\angle -45^\circ = (Z_{th} - j) \times 10\angle -45^\circ$$

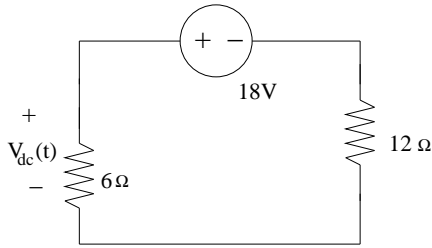
This can be manipulated around to

$$Z_{th} = 2j + 1$$

and then we can plug back in to either of those original equations to obtain

$$V_{oc} = 10\sqrt{2}\angle 0^\circ$$

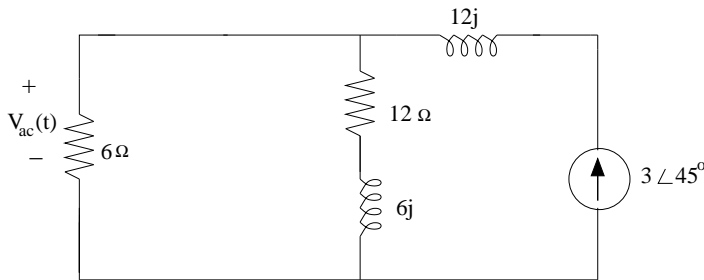
Problem 7: Because we have a DC source and an AC source, we will need to use superposition. First consider the DC source. For DC, the inductors act like short circuits, and the capacitor acts like an open circuit. The AC current source also acts like an open circuit because we are zeroing it out. All of this allows us to simplify the circuit as follows:



Now we can say that the voltage across the 6Ω resistor on the left is, by voltage division:

$$V_{dc} = \frac{18 \times 6}{6 + 12} = 6V$$

Next, we consider the AC source. We need to zero out the DC voltage source, which means it gets replaced by a wire. Hence the capacitor is being shorted out. There can be no voltage difference across the capacitor; we can just replace the parallel combination of the wire and capacitor with a wire. The AC circuit looks like this:



The voltage across 6Ω resistor is therefore:

$$V_{ac} = \frac{(12 + 6j)(6)}{12 + 6j + 6} \times I_s$$

where I_s is the current source phasor: 3 at an angle of 45° .

After some manipulation:

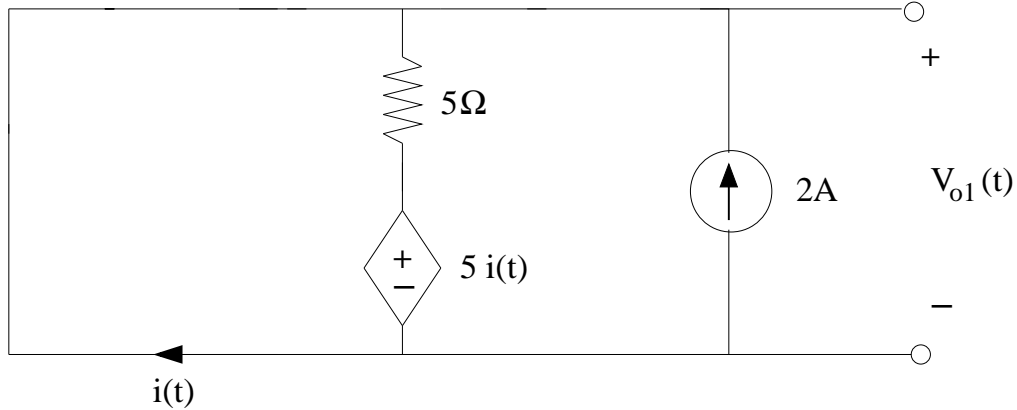
$$V_{ac} = (42 + 6j) \times I_s = 4.243\angle 8.13^\circ \times 3\angle 45^\circ = 12.73\angle 53.13^\circ$$

so $V_{ac}(t) = 12.73 \cos(3t + 53.13^\circ)$.

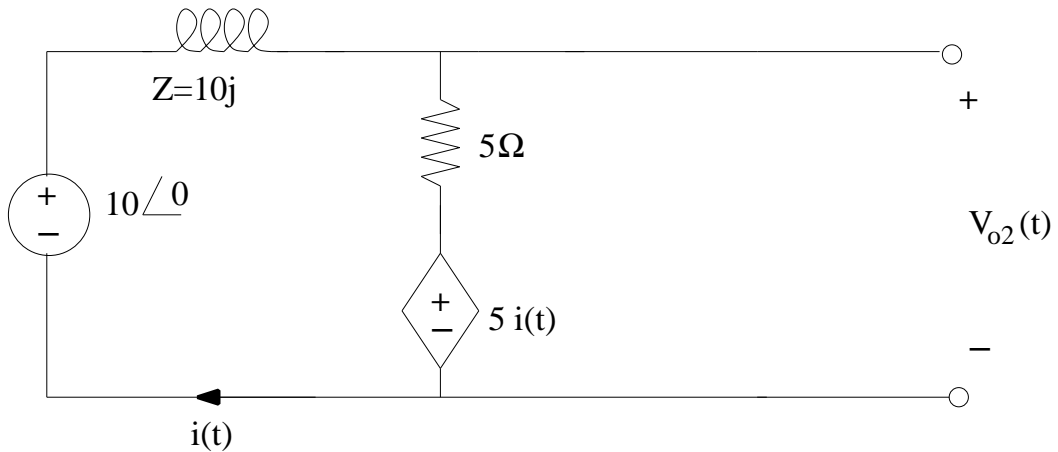
Putting them together:

$$V(t) = V_{dc} + V_{ac}(t) = 6 + 12.73\cos(3t + 53.13^\circ)$$

Problem 8: Again we have an AC voltage source and a DC current source, so we use superposition. Consider first the contribution from the DC current source. To zero out the voltage source, we replace it by a wire (short circuit). At DC, the inductor behaves like a short circuit as well. Therefore the left branch of the circuit is just a wire:



and so the contribution from the DC current source is $V_{o1} = 0V$. Consider now the contribution from the AC voltage source. We zero out the current source by making it an open circuit, and we can also relabel the inductor with its impedance value for this voltage source:



Using KVL:

$$10 = (5 + 5 + 10j)I \qquad I = \frac{1}{\sqrt{2}}\angle -45^\circ \approx 0.707\angle -45^\circ$$

$$V_{o2} = 5I + 5I = 7.07\angle -45^\circ$$

$$V_o(t) = V_{o2}(t) = 7.07\cos(1000t - 45^\circ)$$