HOMEWORK 5: Sampling and Quantization

Due Wednesday, November 26 by start of class

1. **2-D Sampling and Aliasing:**

   a) Write down the 2-D continuous Fourier transform of $\cos(2\pi(u_0x + v_0y))$

   b) Try the following in matlab:

```
[x y] = meshgrid(0:256,0:256);
z = cos(2 * pi * 1/32 .* x - 2 * pi * 1/128 .* y);
z1 = cos(2 * pi * 1/4 .* x -2 * pi * 7/8 .* y);
z2 = cos(2 * pi * 1/2 .* x - 2 * pi * 1/2 .* y);
```

   Look at the images (matrices) x, y, z, z1, z2 using imshow. The meshgrid function has the effect of sampling the three underlying continuous functions with sampling period T = 1 in both x and y directions. Discuss the appearances of the three sampled images. Are the underlying continuous functions getting undersampled? critically sampled (sampled at Nyquist)? oversampled? Explain what you see in terms of the spatial frequencies and the sampling frequency.

   c) What values would we have to choose for `a` and `b` in the expression

```
z3 = cos(2 * pi * a .* x - 2 * pi * b .* y);
```

   so that the sampled function would be aliased and would have an appearance identical to that of the sampled and displayed function `z` ?

2. **2-D DFTs**

   Generate the DFT of the 128 by 128 lena image by typing the following:

```
ln = imread('lena128.tif','tif');
lnfft = fft2(ln);
dsp1 = log(abs(lnfft) + 1);
```

```
dsp2 = fftshift(dsp1);
imshow(dsp1/max(max(dsp1)));
imshow(dsp2/max(max(dsp2)));
```

a) Where is the DC component located in the image dsp1? How can you check that? Where is the DC component located in the image dsp2? What does the function fftshift do? Why is the log used for dsp1? Try displaying abs(lnfft) without the log.

b) Use the circular shift function "ipcircshift2" to generate a circularly shifted version of lena. Take the DFT of this image. Look at the magnitude. Are there any differences between this DFT and the one above? Explain.

3. **Reducing Aliasing:**

Aliasing can be analyzed by finding the foldover frequency, and looking at the area under the folded-over curve. Prefiltering reduces the aliasing power, but it also reduces the signal power in the part of the spectrum we would like to keep intact.

We wish to analyze this reduction in signal power and noise power that comes from pre-filtering. Use the following commands to generate two versions of the lena image:

```
lena = imread('lena512.tif','tif');
lena64_1 = ipsubsample2(lena,8);
lnlow = filter2(ones(8)/64,lena);
lena64_2 = ipsubsample2(lnlow,8);
```

a) Compare visually the two 64×64 versions. Comment on the differences and explain them. Try the same thing with the baboon image (**baboon512.tif**). Notice that the baboon image has a lot more high frequency content than the lena image. Compare your results against those from lena. In particular, address whether the pre-filtering helps more for lena or for baboon.

b) The power spectrum is given by the square of the magnitude of the Fourier Transform. So, the power spectrum for the original lena image is given by

```
ln_fft_1 = fftshift(abs(fft2(lena)).^2);
```

Assume that the underlying continuous image field was sampled at the Nyquist rate to form the original 512×512 discrete image. So the sampling distance is 1, and the sampling frequency is 1, which is the critical sampling frequency.

Since we are downsampling by a factor of 8, that corresponds to what sampling distance? and to what sampling frequency? So what portion of the spectrum is aliased and what portion is not?

For the subsampling from 512 x 512 down to size 64 x 64, estimate the reduction in aliasing power and the reduction in signal power that come from the prefiltering.

4. **Non-rectangular Sampling**

   An image has two-dimensional spectrum with baseband confined to the region where $2 \mid f_1 \mid + \mid f_2 \mid < 2$. Find a lattice in the spatial domain for sampling this image which has the lowest sampling density possible without introducing aliasing.

5. **Scalar Quantization – optimality conditions**

   Suppose that a random variable X has the two-sided exponential pdf

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$$

   A three level quantizer q for X has the form

$$q(x) = \begin{cases} +b & x > a \\ 0 & -a \leq x \leq +a \\ -b & x < -a \end{cases}$$

   (a) Find a simple expression for b as a function of a so that the centroid condition is met.

   (b) For what value of a will the quantizer (using b chosen as above) satisfy the nearest neighbor condition for optimality?