HOMEWORK 4

Due Friday, November 7 by start of class

(But some people have midterms at the end of next week, so you can turn the homework in on Monday the 10th if you need to.)

The image coke.tif is a fairly clean image of a Coke can. The image tran.tif is a version that has been subjected to a pure translation, with no other corruption of the image.

1. Write a routine that implements the Fourier phase correlation method for finding translation parameters. After you inverse transform, you can find the maximum of the array (named out) by using

   ```
   [i,j] = find(out == max(max(out)))
   ```

   Use your routine to register the Coke image to itself. Does the array that comes from the inverse transform look like a perfect discrete delta? You can see what it looks like using matlab's *mesh* command. Report and explain your results.

2. Now use the routine to register the Coke image to the tran image. Again, report and explain your results, and comment on whether or not (and why) the array looks like a perfect discrete delta. Try it both ways: registering coke to tran, and registering tran to coke. What is the best registration?

3. Now we will briefly look at whether this method can handle noise. If the image is scaled between 0 and 255, say, then you can add a significant component of random noise to the translated image as follows:

   ```
   noisy = tran + 40 * randn(425,227);
   ```

   (Check to see that your image is scaled 0 to 255, because if it is scaled between 0 and 1, this would be rather too much noise.) Use your routine to register the Coke image and the noisy image. Discuss the results.

4. Now, try blurring the translated image:

```
filt = hamming(9) * hamming(9)'
filt = filt / sum(sum(filt));
blurred = filter2(filt,tran);
```

Use your routine to register the Coke image and the blurred image, and discuss the results.

5. This time, we will both blur it and add noise:

```
realbad = blurred + 40 * randn(425,227);
```

How does the phase correlation method do now?

6. Try doing something drastic to the tran.tif image, and see whether you can still register the original coke image to your drastically re-worked tran image. For example, you can take a block that is, say 20% of the image area, and swap its location with that of another large block in the image. Or replace a piece with a portion of an unrelated image, or take the square root of the pixel values or whatever. Do you think they can still be registered with the Fourier approach? Try it. What happens?

7. The rationale for the phase correlation method came from the equation:

$$F_2(u,v) = e^{-j2\pi(ux_0+vy_0)}F_1(u,v)$$

where we then multiplied both sides by $F_1^*(u,v)$, and took the magnitude of both sides, and eventually concluded that

$$\frac{F_2(u,v)F_1^*(u,v)}{|F_2(u,v)F_1^*(u,v)|} = e^{-j2\pi(ux_0+vy_0)}$$

which then leads to the idea of taking the inverse transform in order to get a delta function which tells us the translation vector (x_0, y_0). But (as will be discussed further on Monday in class) the equation $F_2(u,v) = e^{-j2\pi(ux_0+vy_0)}F_1(u,v)$ suggests a simpler way to isolate the term $e^{-j2\pi(ux_0+vy_0)}$. One can just do:

$$\frac{F_2(u,v)}{F_1(u,v)} = e^{-j2\pi(ux_0+vy_0)}$$

and use the inverse transform of that ratio to find the delta function that gives us the unknown translation vector (x_0, y_0). Try using this simpler equation to do the registration for the cases above (coke/tran, coke/noisy, coke/blurred, coke/realbad) as well as whatever drastic thing you tried in the previous problem. How does this approach compare to the “regular” phase correlation approach?