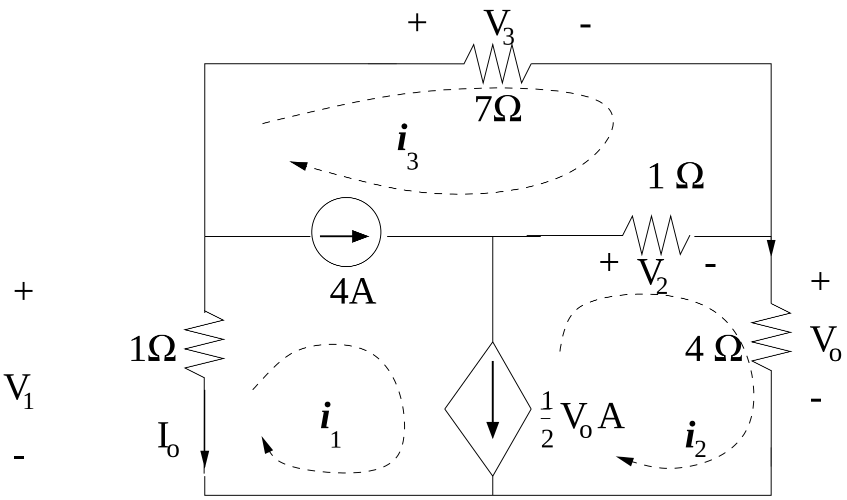
ECE 53a	Homework #2 SOLUTIONS	Instructor: Pamela Cosman	1/28/08

Problem 1: Well, you were told to use mesh analysis, but on closer inspection, this circuit is too simple for that! It is just a voltage divider.

We have to notice that V_o has the opposite polarity from $V_a = 8V$, so we need to include a minus sign in the voltage division equation:

$$V_o = -\frac{2}{2+3+3} \times 8 = -2V$$

Problem 2: A mesh analysis begins with placing mesh currents in each "window pane" and labeling voltages across any circuit element which doesn't already have a voltage named there:



If there were no current sources, what we would do next is to write a KVL equation for each mesh. But, since there are current sources, we write instead the "constraint equations" which come from the fact that a current source constrains the mesh currents that pass through it.

The independent current source of value 4A leads to the constraint:

$$i_1 - i_3 = 4$$

The dependent current source of value $0.5V_o$ leads to:

$$i_1 - i_2 = 0.5V_o$$

And since, by Ohm's Law

$$V_o = 4i_2$$

this equation becomes:

$$i_1 - i_2 = 0.5 \times 4i_2 \qquad\qquad \rightarrow \qquad\qquad i_1 = 3i_2$$

Because we have two constraint equations, we need only one KVL equation. If we write a KVL around the “supermesh” consisting of the entire circuit, we can avoid all the current sources.

$$V_1 - V_3 - V_o = 0$$

Ohm’s Law for these voltages says:

$$V_1 = -i_1 \qquad\qquad V_3 = 7i_3 \qquad\qquad V_o = 4i_2$$

So we substitute into the KVL equation to get:

$$-i_1 - 7i_3 - 4i_2 = 0$$

We can use the two constraint equations to write everything in terms of i_1:

$$-i_1 - 7(i_1 - 4) - 4\frac{i_1}{3} = 0$$

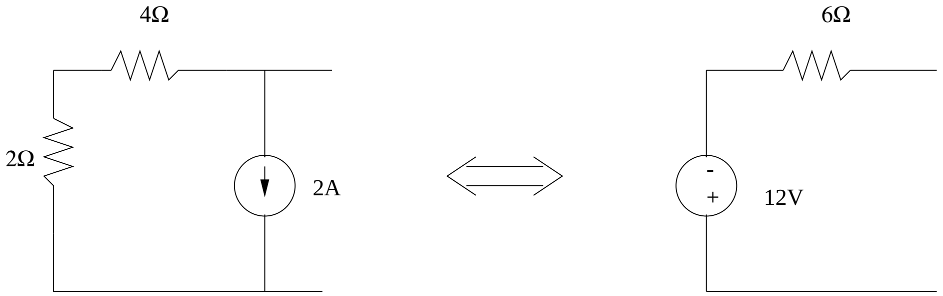
$$-8i_1 + 28 - \frac{4i_1}{3} = 0$$

$$28i_1 = 84 \qquad\qquad \rightarrow \qquad\qquad i_1 = 3A$$

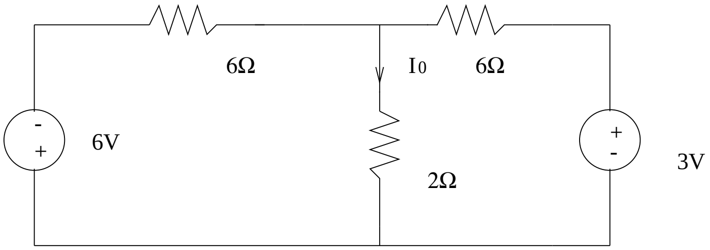
We were asked to find the branch current I_0. The mesh current i_1 is the only mesh current which flows in that branch. But it goes in the opposite direction from I_0. So,

$$I_0 = -3A$$

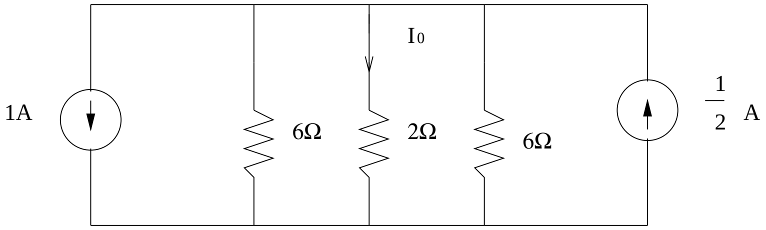
Problem 3: Starting from the left



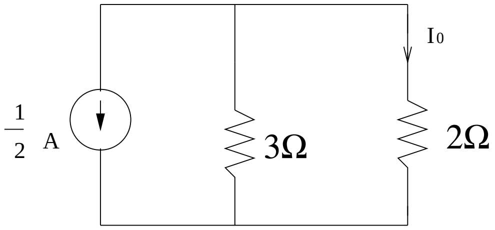
This 12V source can be combined with the 6V source of the opposite polarity to produce:



Now, let's do source transformation on both sides of the I_0 branch.



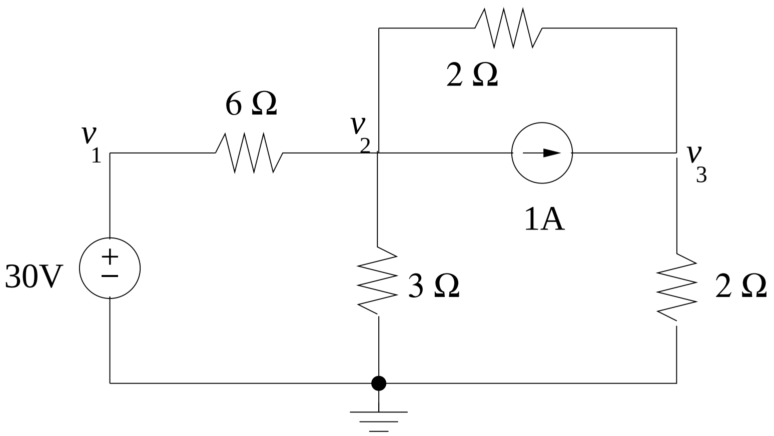
The two current sources in parallel can be combined, and the two 6Ω resistors in parallel can also be combined (they are equivalent to a 3Ω resistor). The circuit now looks like this:



By current division we can say that

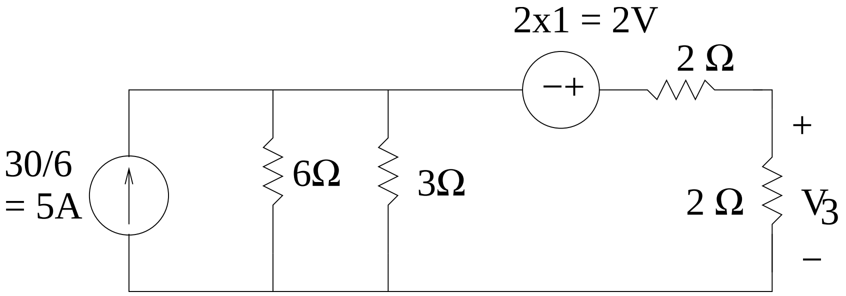
$$I_0 = \frac{-\frac{1}{2} \times 3}{3+2} = -\frac{3}{10}A$$

Problem 4: The starting circuit looks like this:

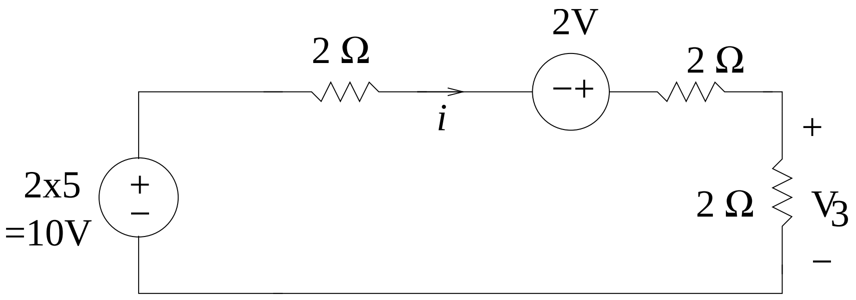


We can make 2 source transformations on this at the same time. On the left hand side, the 30V voltage source and the series 6Ω resistor can get transformed into a current source with a parallel resistor.

On the top, the current source with the 2Ω resistor can get transformed into a voltage source with a series resistor. The new circuit looks like this:



The 6Ω and 3Ω resistors in parallel can get combined, and they are equivalent to a 2Ω resistor. This 2Ω resistor together with the current source on the left can get transformed back into a voltage source with a series resistor:

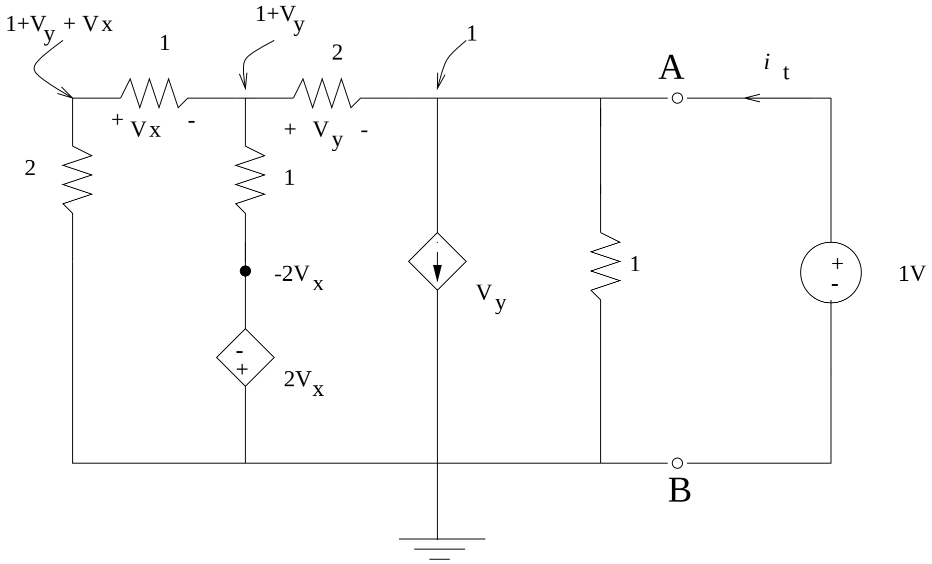


At this point, the circuit has only one loop. We can solve for the voltage V_3 as follows:

$$10 + 2 = 2i + 2i + 2i \qquad \text{and so} \qquad i = 2A \qquad \text{and so} \qquad V_3 = 2 \times 2 = 4V$$

Problem 5: Here there are only dependent sources, so the Norton (or Thevenin) equivalent circuit will just be a resistor. To determine the value of this resistor, we have to attach an external test source. I will attach a 1V voltage source, and solve for the current using nodal analysis. The problem can also be done using an external current source, instead of a voltage source. Note that the value of the external source does not need to be specified, and it you do choose to specify a value, it can be arbitrary, since the only thing that will matter is the ratio of the external voltage to current.

Using a nodal analysis, we can label the nodes along the top directly as having values of 1, $1 + V_y$ and $1 + V_y + V_x$.



Using KCL at node $(1 + V_y)$:

$$\frac{V_y}{2} + \frac{(1 + V_y) - (-2V_x)}{1} + \frac{-V_x}{1} = 0$$

Using KCL at node $(1 + V_y + V_x)$, we can equate the current that goes downward through that 2Ω resistor, and the current that flows into the node from the right.

$$\frac{1 + V_y + V_x}{2} = \frac{-V_x}{1}$$

Now we have two equations in two unknowns, and we can solve for V_x and V_y.

In the second equation, we multiply through by 2 and re-arrange terms to obtain

$$V_y = -1 - 3V_x$$

In the first equation, we multiply through by 2 to obtain

$$V_y + 2 + 2V_y + 4V_x - 2V_x = 0$$

$$3V_y + 2 + 2V_x = 0$$

Now we can substitute for V_y:

$$3(-1 - 3V_x) + 2 + 2V_x = 0 \qquad \Rightarrow \qquad V_x = \frac{-1}{7}$$

Therefore

$$V_y = -1 - 3\left(\frac{-1}{7}\right) = \frac{-4}{7}$$

Now we can get the test current i_t by writing a KCL equation at node A:

$$i_t = 1 + V_y - \frac{V_y}{2} = 1 + \frac{V_y}{2} = 1 - \frac{2}{7} = \frac{5}{7}$$

So in the end, the equivalent resistance is

$$R_0 = \frac{V_t}{i_t} = \frac{1}{5/7} = \frac{7}{5} = 1.4A$$