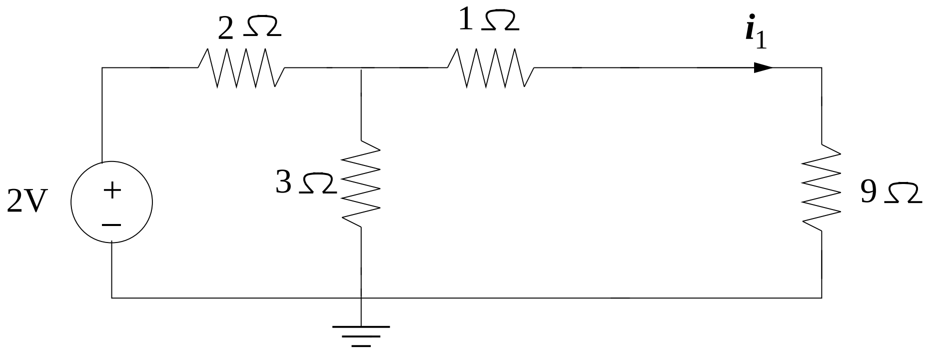
Problem 1: To find the current due to the 2V voltage source, we zero out the current source by making it an open circuit:



The resistance loading the voltage source is

$$R = 2 + \frac{3 \times 10}{3 + 10} = \frac{56}{13}$$

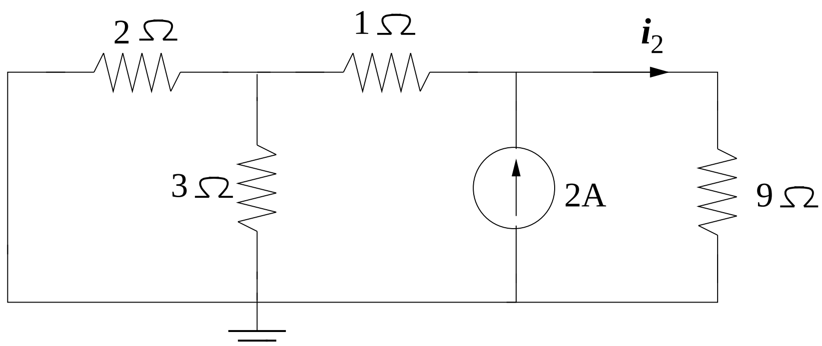
and so the source current is

$$i_s = \frac{2}{R} = \frac{26}{56}$$

By current division,

$$i_1 = \frac{i_s 3}{3 + 10} = \frac{6}{56} = \frac{3}{28} \approx 0.107$$

To find the current due to the 2A current source, we zero out the voltage source by making it a short circuit:



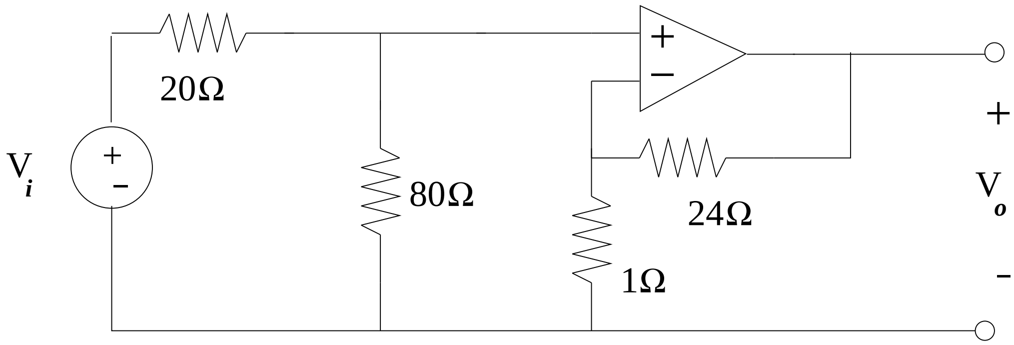
To the left of the current source, 2 in parallel with 3 is 6/5, and that's in series with 1, so that resistance is 11/5. By current division,

$$i_2 = \frac{11/5}{11/5 + 9} \times 2 = \frac{11}{28} \approx 0.393$$

The total current is

$$i = i_1 + i_2 = \frac{3}{28} + \frac{11}{28} = \frac{14}{28} = 0.5$$

Problem 2: (8 points)



Using voltage division,

$$V_+ = V_i \frac{80}{20+80} = 0.8V_i$$

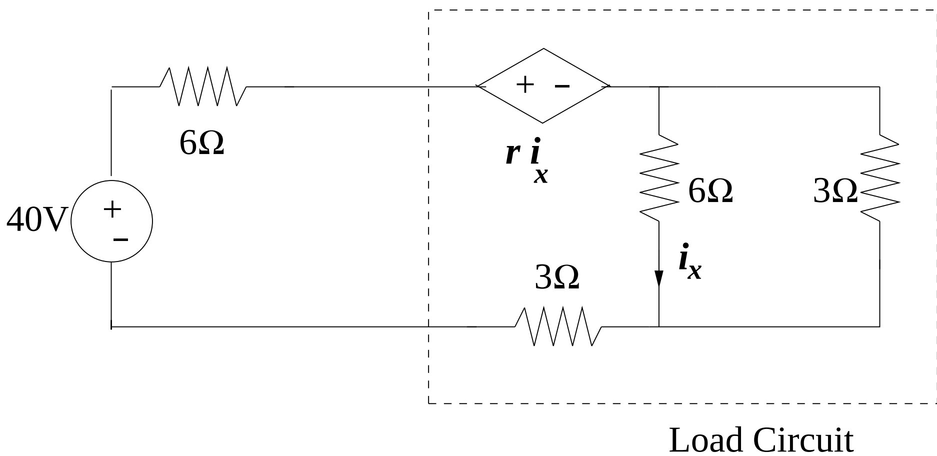
also

$$V_+ = V_- = V_o \frac{1}{1+24} = 0.04V_o$$

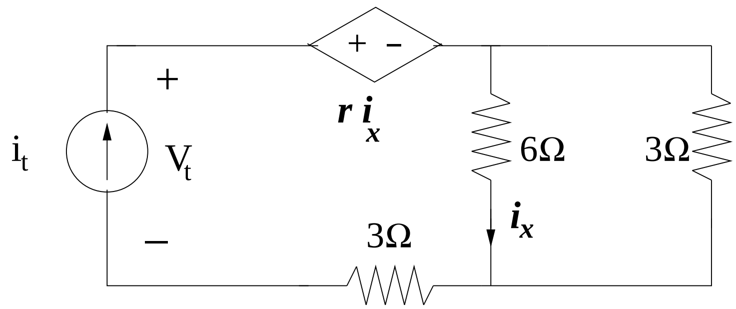
so

$$\frac{V_o}{V_i} = \frac{0.8}{0.04} = 20$$

Problem 3: (12 points)
In the following circuit, the value of r that will maximize the power delivered to the load circuit is when r is equal to the Thevenin equivalent resistance, which is 6Ω.



The load circuit is equivalent to some resistor, since it has no independent sources. Let us find what resistance it is equivalent to by hooking it up to a test current source:



By current division,

$$i_x = \frac{i_t}{3}$$

Writing KVL around the left-hand loop and then substituting for i_x:

$$V_t = 3i_t + 6i_x + ri_x = 3i_t + \frac{6+r}{3}i_t$$

$$\frac{V_t}{i_t} = 3 + \frac{6+r}{3}$$

This needs to be equated to 6:

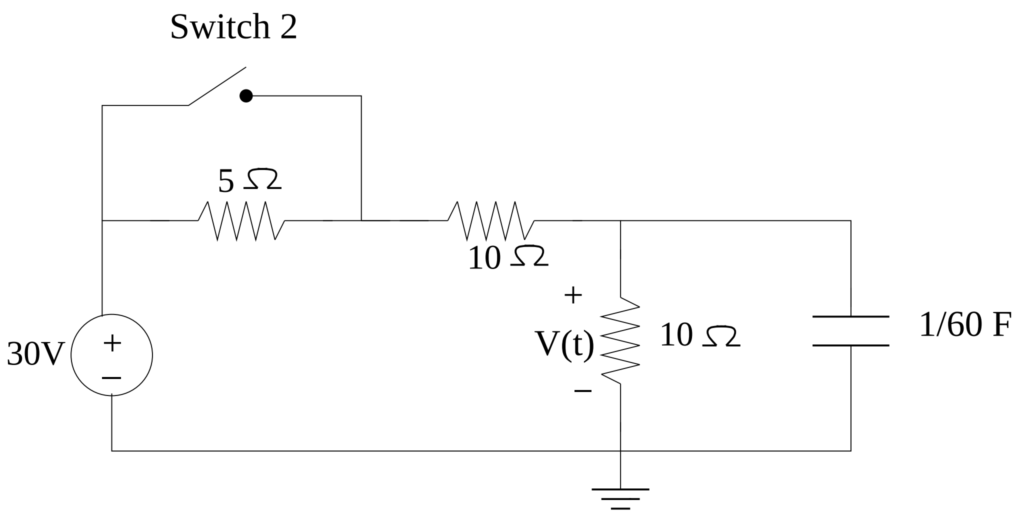
$$3 + \frac{6+r}{3} = 6 \rightarrow r = 3$$

Problem 4: (a) (2 points) Both Switches have been open for a very long time, and Switch 1 is closed at $t = 0$. When switch 1 is closed, the 30V source is directly across the 5Ω resistor. The node between switch 1 and switch 2 becomes part of the ground node. The right hand portion of the circuit is no longer connected to the source. So this is a natural response.

(b) (2 points) Switch 2 has been open for a very long time, and Switch 1 has been closed for a very long time, and then Switch 1 is opened at $t = 0$. When switch 1 is opened, the source is connected to the whole rest of the circuit, so this is a step response.

(c) (2 points) Both Switches have been open for a very long time, and Switch 2 is closed at $t = 0$. Also a step response, for the same reason as above.

(d) (8 points) Determine the voltage $V(t)$ for $t > 0$ for the circuit below, where Switch 2 has been closed for a very long time and opens at $t = 0$.



Before $t = 0$, the capacitor acts as an open circuit to DC, and the switch is closed. So the whole circuit just has a 30V source across two 10 Ω resistors. So $V(0^-) = 15V$.
Since the unknown voltage which we are trying to find is the voltage across a capacitor, the value at $t = 0^+$ is the same as the value at $t = 0^-$. For $t > 0$ we get by KCL:

$$\frac{1}{15}(v - 30) + \frac{1}{10}v + \frac{1}{60}\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + 10v = 120$$

$$v = 12 + 3e^{-10t} Volts$$

Problem 5: (8 points) Since Z has a resistive component, one of the impedances must be a resistor. Let $Z_1 = R$. The other one must be a capacitor, since the imaginary portion of the impedance is negative. So $Z_2 = 1/j\omega C$. But if you don't see that the other component is a capacitor, you can always just try it both ways (inductor, capacitor) and see which one works.
The parallel combination of Z_1 and Z_2 has to be $5 - j5$

$$\frac{\frac{1}{j\omega C}R}{R + \frac{1}{j\omega C}} = 5 - j5$$

Multiplying the left hand side, top and bottom, by $j\omega C$ we get

$$\frac{R}{Rj\omega C + 1} = 5 - j5$$

$$R = 5Rj\omega C + 5 - j5 + 5R\omega C$$

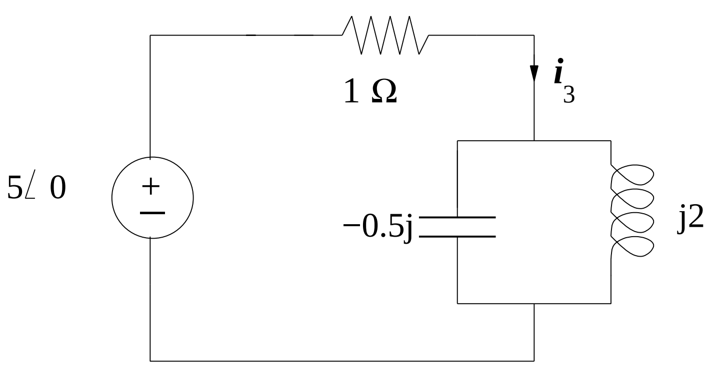
which gives us two equations, one for the real part and one for the imaginary part:

$$5R\omega C - 5 = 0 \qquad \text{and} \qquad R = 5 + 5R\omega C$$

from which we get $R = 10\Omega$ and

$$C = \frac{1}{10\omega} = \frac{1}{102\pi 60} = 265\mu F$$

Problem 6: (15 points) This problem must be solved by superposition. Keeping just the DC source, the circuit has only 1 Ω resistor connected across a DC source, so $i_1 = -2.5A$.
Keeping the $\omega = 1$ source, the inductor behaves like $j\omega L = j$ and the capacitor behaves like $1/j\omega C = -j$. And, just like we did in class during the last lecture, an impedance of j in parallel with an impedance of -j is infinite impedance, so the current is zero. $i_2 = 0$
Lastly, keeping the $\omega = 2$ source, the circuit becomes:



$$Z = 1 + (j2)||(-j0.5) = 1.2\angle - 33.7^o$$

$$I_3 = \frac{5}{1.2\angle - 33.7^o} = 4.17\angle 33.7^o$$

So the final answer is the superposition:

$$i(t) = 4.17\sin(2t + 37^o) - 2.5A$$

Problem 7: (6 points)

$$I_{eff} = \sqrt{\frac{1}{2}\int_0^1 2^2 dt + \int_1^2 (-1)^2 dt} = \sqrt{\frac{1}{2}(4+1)} = \sqrt{\frac{5}{2}} = 1.58$$

Problem 8: (15 points) (a) The voltage across the inductor is

$$V_L(t) = L\frac{di}{dt} = 10^{-2} \times \frac{0.1A}{10^{-3}s} = 1V$$

(b) Assuming no initial charge on the capacitor, its voltage is

$$V_C(t) = 0 + \frac{1}{10^{-5}} \int_0^t 100x dx = \frac{10^7}{2} t^2$$

(c) The stored energy in the capacitor is $\frac{1}{2}CV^2$ and the stored energy in the inductor is $\frac{1}{2}LI^2$. So

$$\frac{1}{2} 10^{-5} \left(\frac{10^7 t^2}{2} \right)^2 > \frac{1}{2} 10^{-2} (100t)^2 \rightarrow t > 0.632msec$$

Problem 9: (14 points) By KCL at the inverting input of the op amp

$$\frac{V_i}{R} + \frac{V_o}{R} + \frac{V_o}{1/j\omega C} = 0$$

This can be massaged around to give us:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{-1}{1 + j\omega RC}$$

Thus we have

$$|H| = \frac{1}{\sqrt{(1 + (\omega RC)^2)}}$$

This is exactly the amplitude response of the first-order RC lowpass filter which we discussed in class, where it starts at 1 (for $\omega = 0$) and goes down to 0 (as $\omega \rightarrow \infty$) and passes through $1/\sqrt{2}$ at the half power frequency $\omega = 1/RC$.