ECE 53a	FINAL EXAM Solutions	Pamela Cosman	3/17/08

Problem 1: We begin by considering the circuit before time t=0, when it is in DC steady state with the switch open. The inductor can be replaced by a short circuit. So, for $t < 0$, the voltage across the inductor is zero. The circuit reduces to a 9A current source with two resistors in parallel, so the current through the inductor can be found by current division:

$$i_L(t = 0^-) = \frac{9 \times 12}{12 + 6} = 6A = i_L(t = 0^+)$$

At time 0, the switch closes, effectively shorting out everything to the left of it. So the circuit at this point can be treated as just a 3H inductor and a 6Ω resistor. The time constant is

$$\tau = \frac{L}{R} = \frac{3}{6} = 0.5$$

So the current will decay as

$$i_L(t) = i_L(t = 0^+)e^{-t/\tau} = 6e^{-2t}$$

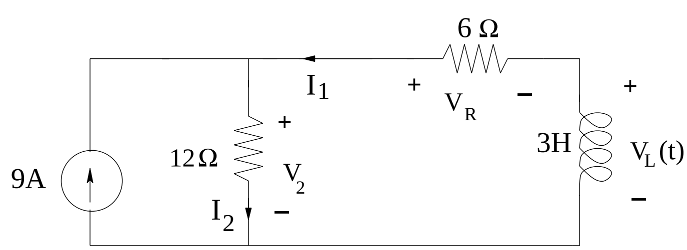
We are asked for the voltage however. The initial condition on the voltage can be found by considering that the initial current of 6A goes (from left to right) across the 6Ω resistor, producing a voltage drop of 36V. So

$$V_L(t) = V_L(t = 0^+)e^{-t/\tau} = -36e^{-2t}$$

If the switch did not flip again, then this would be the answer for all $t > 0$. But because the switch does flip again, this expression is valid only for $0 < t < 1$. We evaluate the situation at $t = 1$.

$$i_L(t = 1) = 6e^{-2} \approx 0.812$$

At this point, the switch opens again, and the situation at time $t = 0^+$ is this:



where I_1 = -0.812 and I_2 can be found from KCL to be:

$$I_2 = 9 + I_1 = 9 - 0.812 = 8.188A$$

which means that, using KVL: $V_2 = V_R + V_L$
where

$$V_2 = 12 \times I_2 = 12 \times 8.2 = 98.256V$$

$$V_R = -6 \times I_1 = 6 \times 0.812 = 4.872V$$

which leads to

$$V_L(t = 1^+) = V_2 - V_R = 98.256 - 4.87 = 93.386V$$

The time constant of the circuit, with the switch open is:

$$\tau = \frac{L}{12 + 6} = \frac{3}{18} = \frac{1}{6}$$

As time goes towards infinity, the circuit will settle into a new DC steady state, in which the inductor again acts like a short circuit, so the voltage across it must go to zero. So for time after 1 sec:

$$V_L(t) = 93.386e^{-6(t-1)}$$

Problem 2: Two of the four missing values should have been easy. These are worth 2 points each.
• When the load resistance is zero, there can be no voltage sustained across it. The voltage is zero.
• When the load resistance is infinite, there can be no current through it. The current is zero.

• The middle line is easy too. This is also worth 2 points. By Ohm's Law, if the voltage across a 200Ω resistor is 10 V, then the current through it is

$$I_L = \frac{V_L}{R_L} = \frac{10}{200} = 0.05A$$

• The only tricky item is the voltage across the infinite resistance. This is worth 4 points. Here we need to invoke a Thevenin or Norton equivalent model for the unknown circuit.
I will use a Norton model. The short circuit current is given as $I_{sc} = 0.15$ A. The Norton resistance R_0 in parallel with the current source can be calculated from the fact that when 10 V appears across the load, 0.05 A flows in the load. Therefore, when 10V are across the Norton parallel resistance R_0, there are $0.15 - 0.05$ A going across it.

$$R_0 = \frac{10}{0.15 - 0.05} = \frac{10}{0.1} = 100\Omega$$

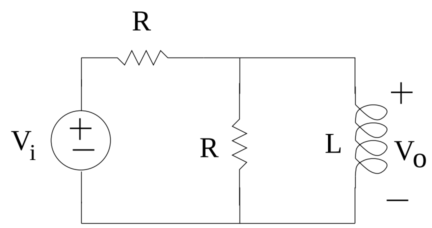
Now we know that when the load resistor is infinite, the 0.15 current source will have nowhere else to go but across the 100 Ω resistor R_0, producing an open circuit voltage of

$$V_{oc} = 0.15 \times 100 = 15V$$

And the final table looks like this:

V_L	I_L	R_L
0 V	0.15 A	0
10 V	0.05 A	200 Ω
15 V	0 A	∞

Problem 3: I did a lowpass filter circuit in class on 3/10 that was like this. Now this is the same thing but for highpass. The circuit we need is:



The transfer function is:

$$H(j\omega) = \frac{R||j\omega L}{R + R||j\omega L} = \frac{\frac{Rj\omega L}{R+j\omega L}}{R + \frac{Rj\omega L}{R+j\omega L}} = \frac{Rj\omega L}{R(R+j\omega L) + Rj\omega L} = \frac{j\omega L}{R + 2j\omega L} = \frac{j\omega L/R}{1 + 2j\omega L/R}$$

The magnitude is

$$|H(j\omega)| = \frac{\omega L/R}{\sqrt{1 + \frac{4\omega^2 L^2}{R^2}}}$$

We can see this is the sort of highpass filter we wanted, because as $\omega \to 0$, $|H| \to 0$ (which is an essential requirement for this to be a highpass filter) and also as $\omega \to \infty$, $|H| \to 1/2$ (which satisfies the condition on the maximum magnitude being 1/2).
The passband extends up to f=20kHz, which corresponds to $\omega_c = 2\pi f = 20,000 \times 2\pi$.
So that is the halfpower frequency, and we can write:

$$|H(j\omega_c)| = \frac{1}{2\sqrt{2}} = \frac{20,000 \times 2\pi L/R}{\sqrt{1 + \frac{4(20,000\times 2\pi)^2 L^2}{R^2}}}$$

We need to choose L and R so that this equation comes out true. So, we can choose one of them arbitrarily. Let's choose $R = 20,000 \times 2\pi$ just to make the equation simpler. Then

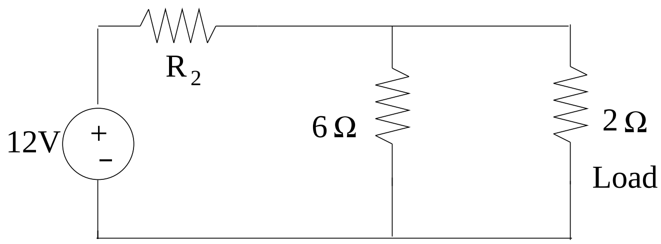
$$\frac{1}{2\sqrt{2}} = \frac{L}{1 + 4L^2}$$

$$\sqrt{1 + 4L^2} = 2\sqrt{2}L$$

$$1 + 4L^2 = 8L^2$$

which leads to $L = 1/2$. Of course there exists an infinite set of correct solutions for R and L; they just have to satisfy the equation for halfpower frequency.

Problem 4: The load circuit is just a 6Ω resistor in parallel with a 3Ω resistor which is equivalent to a 2Ω resistor. (a) The circuit for part (a) is



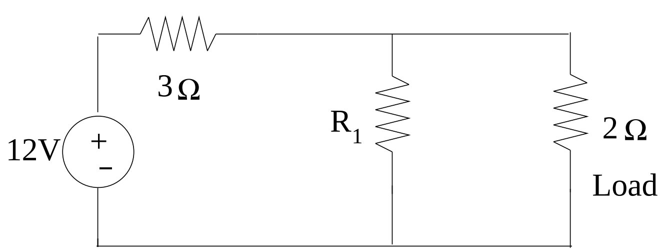
It should be obvious by inspection that R_2 should be chosen to be zero, to maximize the power absorbed by the load. The voltage of the source is dividing between the R_2 resistor and the parallel combination of the other (which we can't change). Any non-zero value of the R_2 resistor will cause some voltage drop across it, thereby diminishing the voltage drop across the load, and diminishing the power absorbed. If this is not obvious, one can write an equation, for example:

$$P = \frac{1}{2}\frac{V^2}{2}$$

where V is the voltage across the load, equal to

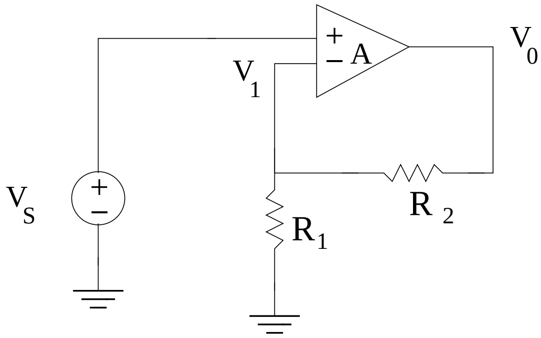
$$V = \frac{12 \times 1.5}{R_2 + 1.5} = \frac{18}{R_2 + 1.5}$$

because the parallel combination of 6 and 2 is 1.5. At this point, we see that to maximize P we need to maximize V, which means to take $R_2 = 0$.
(b) The circuit for part (b) is



It should be obvious by inspection that R_1 should be chosen to be infinite, to maximize the power absorbed by the load. Again if it is not obvious, one can write an equation for either the current through the load or the voltage across the load, and show that the function takes on its maximum value for $R_1 = \infty$.

Problem 5: The circuit for the non-inverting amplifier looks like this:



We are told that the current in the feedback circuit should be 0.1mA when the output voltage is 1V. The equation for this is:

$$i = \frac{V_0}{R_1 + R_2} = \frac{1}{R_1 + R_2} = 10^{-4}$$

This gives us one equation relating R_1 and R_2. To get a second one, we use the fact the overall (closed-loop) voltage gain should be 5.

$$V_0 = A(V_s - V_1) = A(V_s - \frac{V_0 R_1}{R_1 + R_2})$$

$$V_0 + \frac{A V_0 R_1}{R_1 + R_2} = A V_s$$

$$V_0 = \frac{A}{1 + \frac{A R_1}{R_1 + R_2}} \times V_s = 5 V_s$$

So our two equations are:

$$R_1 + R_2 = 10^4$$

$$100 = 5 + \frac{500 R_1}{R_1 + R_2}$$

We solve this to get $R_1 = 1900\Omega$ and $R_2 = 8100\Omega$

Problem 6: For M1, because input A is low, $V_G = 0$. And the source is connected to +5V, so $V_S = 5$. Therefore $V_{GS} = -5V$ which is larger (in magnitude) than the threshold voltage device (this is a PMOS device) so the device is ON. Since the problem statement says to use the switch model, when the device is ON, we consider it to be a wire, so there is no voltage drop across it, so $V_D = 5V$ also, and $V_{DS} = 0V$.

For M2, input B is high, so $V_G = 5$. And the source of this device is connected to the drain of M1, so $V_S = 5$. $V_{GS} = 0V$. This means that this device is OFF. According to the switch model, it behaves like an open circuit. So we can't tell at this point what V_D is.

For M3, input B is high, so $V_G = 5$. And the source of this device is connected to ground, so $V_S = 0$. Therefore $V_{GS} = +5V$ which is larger than the threshold voltage device (this is a NMOS device) so the device is ON. With the switch model, when the device is ON, we consider it to be a wire, so there is no voltage drop across it, so $V_D = 0V$ also, and $V_{DS} = 0V$.

Since the drain for this M3 device is connected by a wire to the drain for the M2 transistor, now we can answer $V_{DS} = 0 - 5 = -5V$ for M2.

Lastly for M4, input A is low, $V_G = 0$, $V_S = 0$ so $V_{GS} = 0$ so the device is OFF. And $V_D = 0$ so $V_{DS} = 0$. So finally the table looks like this:

	V_{GS}	OFF/ON	V_{DS}
M1	-5	ON	0
M2	0	OFF	-5
M3	+5	ON	0
M4	0	OFF	0

(b) For this part, we can make a truth table. From part (a) we already have the case that C=0 when A=0 and B=1. We have to work out the other 3 cases, and we get:

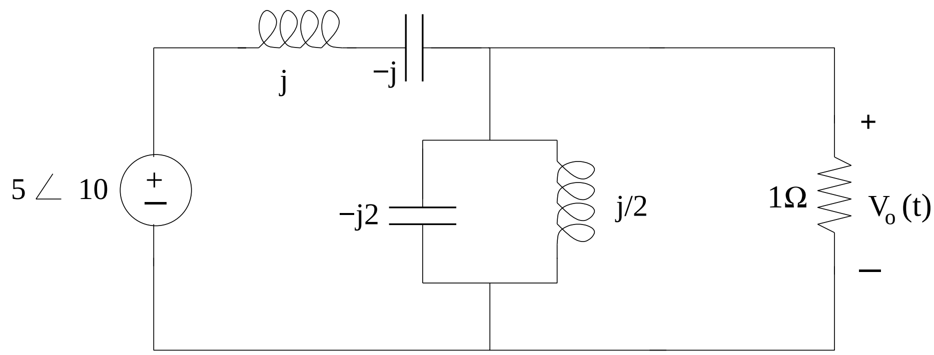
A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

which is a NOR gate.

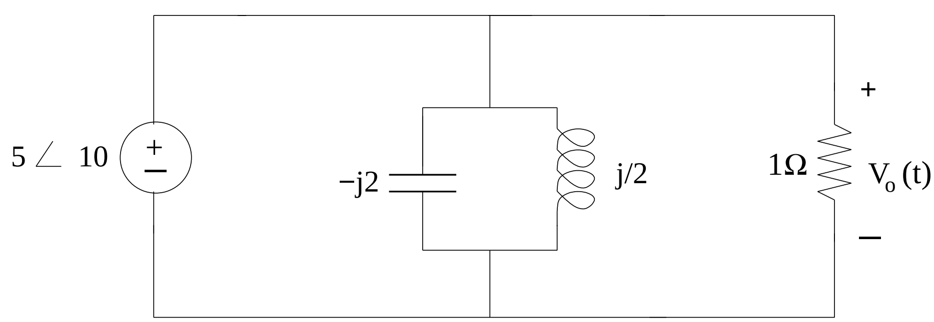
Problem 7: Here there are 3 sources, at difference frequencies, so we have to use superposition. We begin with the DC voltage source. At DC, inductors behave like short circuits, and capacitors behave like open circuits, and so the whole circuit reduces to just the 10V DC source directly across the 1Ω resistor. So the component of the output voltage V_o that comes from the 10 V DC source is

$$V_{DC} = 10V$$

Next consider the AC voltage source. It has frequency $\omega = 1$. So inductors behave like impedances with impedance value $j\omega L = j \times 1 \times L$ and capacitors behave like impedances with impedance value $1/(j\omega C) = 1/(jC) = -j/C$.
So the circuit looks like this, considering the effect of the AC voltage source alone:



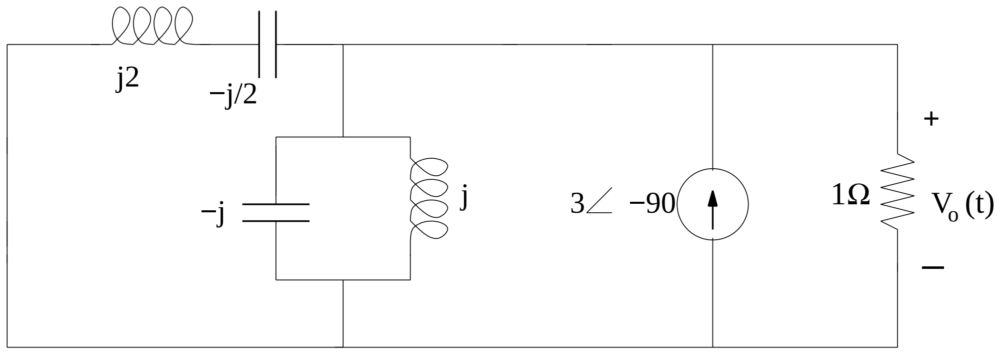
Now the two impedances on the top left are j and $-j$ in series, so that adds up to zero, so the circuit reduces to



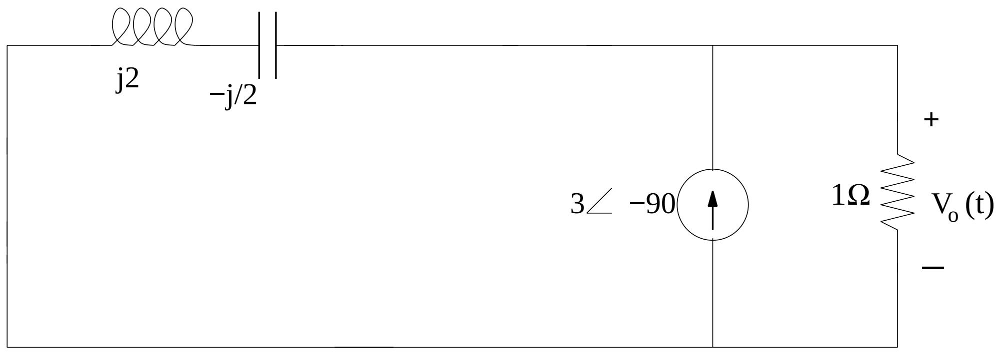
which shows that the AC voltage source is directly across the 1Ω resistor on the right, so the component of the output voltage V_o that comes from the AC voltage source is

$$V_{AC1} = 5\cos(t + 10^o)$$

Lastly, we consider the AC current source. Here, the frequency is $\omega = 2$, and the circuit looks like this:



The parallel combination of j and -j is an infinite impedance, so we can remove that branch from the circuit:



and the voltage can then be found as:

$$V = 3\angle - 90^o \times (1||1.5j) = 2.5\angle - 56^o$$

$$V_{AC2} = 2.5\cos(2t - 56^o)$$

The output voltage is the sum of the 3 terms:

$$V_o(t) = 10 + 5\cos(t + 10^o) + 2.5\cos(2t - 56^o)$$

Problem 8: the parallel combination of the inductor and the resistor has impedance

$$Z_{eq} = \frac{j\omega LR}{j\omega L + R} = \frac{j\omega L}{\frac{j\omega L}{R} + 1}$$

The transfer function can then be written down by voltage division as

$$H(j\omega) = \frac{V_o}{V_i} = \frac{Z_{eq}}{Z_{eq} + \frac{1}{j\omega C}} = \frac{j\omega L}{j\omega L + \frac{1}{j\omega C}[1 + \frac{j\omega L}{R}]} = \frac{(j\omega)^2}{(j\omega)^2 + 10j\omega + 100}$$

(b) The magnitude of this is

$$|H(j\omega)| = \frac{\omega^2}{[(100\omega^2) + (100 - \omega^2)^2]^{-0.5}}$$

(c) As $\omega \to 0$, $|H| \to 0$ because the numerator of this expression goes to zero, but the denominator does not. As $\omega \to \infty$, $|H| \to 1$, because we can ignore everything but the highest term in ω, which is the same for the numerator of this expression and the denominator. So this is a highpass filter.

Problem 9: This is from last year's final, which was given to you for practice.
(a) Since the voltage is $1\angle 0$, the current through the inductor is

$$i_L(t) = \frac{1\angle 0}{6 + j \times 3 \times 2} = \frac{1\angle 0}{\sqrt{72}\angle 45} = \frac{1}{\sqrt{72}}\angle - 45$$

The energy in the inductor depends on the current through the inductor as follows:

$$w_L(t) = \frac{1}{2}Li_L^2(t) = \frac{1}{2} \times 2 \times \frac{1}{72}\cos^2(3t - 45) = \frac{1}{72}\cos^2(3t - 45)$$

(b) The energy stored in the capacitor depends on the voltage across it, which is just $V_1(t)$.

$$w_C(t) = \frac{1}{2}CV_1^2(t) = \frac{1}{2} \times \frac{1}{36}\cos^2(3t) = \frac{1}{72}\cos^2(3t)$$

(c) The average power dissipated by the resistor is

$$P = \frac{1}{2}RI_{max}^2 = \frac{1}{2} \times 6 \times \frac{1}{72} = \frac{1}{24}$$

So the energy dissipated in one cycle is

$$E = TP = \frac{2\pi}{3} \times \frac{1}{24} = \frac{\pi}{36}$$