Problem 1: The Thevenin equivalent circuit inside the black box consists of a voltage source V_{oc} in series with a resistor R_{th}.

When attaching a load resistor R_L, the maximum power will be dissipated in the load if $R_L = R_{th}$. From the plot, we see that the max power dissipation occurs when $R_L = 20\Omega$. Therefore we can conclude that $R_{th} = 20\Omega$.
The max power dissipated is 10 W. We can say:

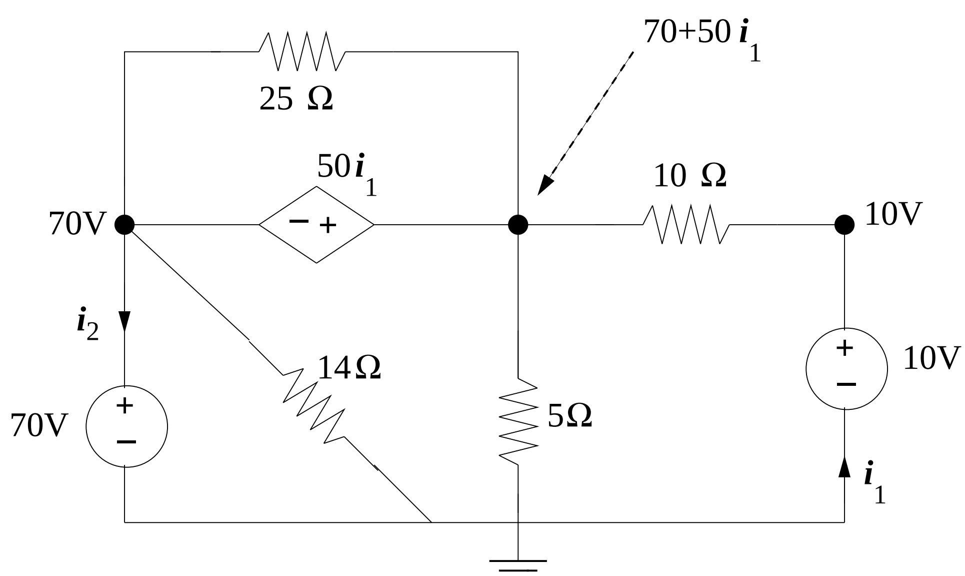
$$10 = R_L I^2$$

where the current I is equal to

$$I = \frac{V_{oc}}{R_{th} + R_L} = \frac{V_{oc}}{2R_L}$$

$$10 = \frac{R_L V_{oc}^2}{4R_L^2} = \frac{V_{oc}^2}{4R_L} = \frac{V_{oc}^2}{80} \quad \Rightarrow \quad V_{oc} = \sqrt{800}$$

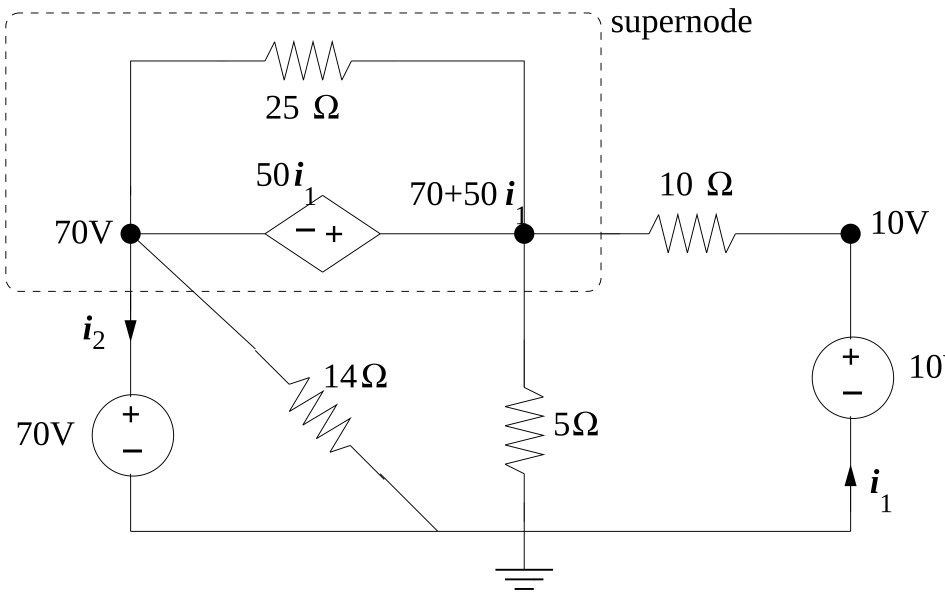
Problem 2: Because there are two voltage sources, this one will be fast with nodal analysis. We put the ground node at the bottom. The left hand node has a value of 70V and the right hand node is 10V. The node in the middle is $70 + 50i_1$ because of the dependent voltage source.



So we can obtain i_1 immediately just by writing Ohm's Law for the 10 Ω resistor:

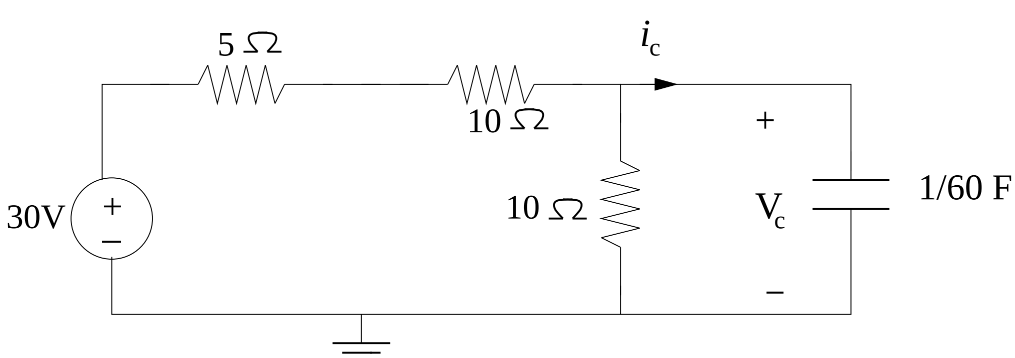
$$\frac{10 - 70 - 50i_1}{10} = i_1 \quad \rightarrow \quad -60 - 50i_1 = 10i_1 \quad \rightarrow \quad -60 = 60i_1 \quad \rightarrow \quad i_1 = -1\text{A}$$

Now, we can get i_2 by writing a single KCL equation for the supernode:



$$i_2 = \frac{0 - 70 - 50i_1}{5} + \frac{0 - 70}{14} - 1 = -4 - 1 - 5 = -10\text{A}$$

Problem 3: Prior to time zero, the switch is closed, which effectively shorts out everything in the circuit to the right of the switch. That is, all voltages and currents in the circuit to the right of the switch are zero. The 30V voltage source is directly across the 5Ω resistor.
So, $V_c(0^-) = V_c(0^+) = V_c(0) = 0\text{V}$
For $t > 0$, the circuit looks like this:



We can write one node voltage (KCL) equation:

$$\frac{30 - V_c}{15} = \frac{V_c}{10} + i_c$$

and then use the capacitor law

$$i_c = C\frac{dV_C(t)}{dt}$$

to write

$$\frac{30 - V_c}{15} = \frac{V_c}{10} + \frac{1}{60}\frac{dV_C(t)}{dt}$$

This can be rearranged to obtain:

$$\frac{dV_C(t)}{dt} + 10V_c = 120$$

The solution is

$$V_c = 12 - 12e^{-10t}$$

to satisfy the initial condition on $V_c(0) = 0$.
So, we can write the expression for v(t) for all time as either

$$v(t) = 0 \text{ for } t \le 0 \text{ and } v(t) = 12 - 12e^{-10t} \text{ for } t \ge 0$$

or as

$$v(t) = (12 - 12e^{-10t})u(t) \text{ for all time}$$

(b) The voltage across the capacitor must be continuous, and this is also the voltage across the resistor. Since the current i_R through a resistor is proportional to the voltage across it, the current i_R must be continuous as well.
The current i_c through the capacitor, on the other hand, does not have to be continuous. And in this case, it has a jump discontinuity. We can see that by saying

$$i_c = C\frac{dV_C(t)}{dt}$$

and so $i_c = 0$ for $t < 0$ and $i_c = \frac{1}{60}120e^{-10t}$ for $t > 0$. So $i_c(0^-) = 0$ and $i_c(0^+) = 2$ so there is a jump discontinuity.

(c) We normally consider that the circuit is back in steady state after five time constants have elapsed.
The time constant for this circuit is $T = \frac{1}{10}$, so at $5T$ or one half second, the effect of throwing the switch has settled down, and we are back in steady state.

Problem 4: Since this is an ideal op amp with negative feedback, we can assume that the two input terminals are at the same voltage. Since one of them is grounded, the other one is at zero volts also. We can write a KCL equation there:

$$\frac{V_1 - 0}{20} + \frac{V_2 - 0}{10} = \frac{0 - V_0}{1/j\omega C}$$

where $V_1 = 80\angle 0$ and $V_2 = 40\angle 0$ and C=1 and $\omega = 100$.
This equation can be re-arranged to:

$$V_1 + 2V_2 = -j2000V_0$$

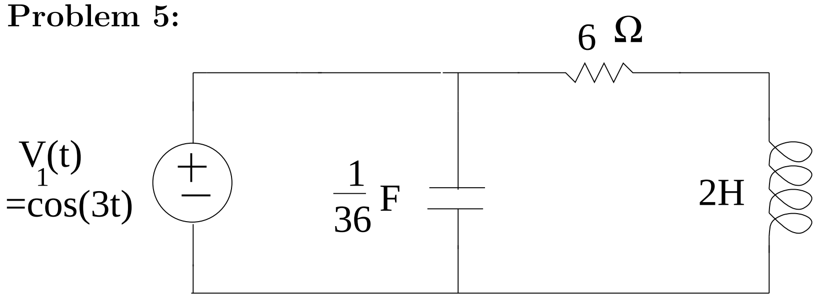
$$160\angle 0 = -j2000V_0$$

$$V_0 = \frac{160\angle 0}{-j2000} = \frac{8j\angle 0}{100} = .08\angle 90^o$$

So

$$V_0(t) = .08\cos(100t + 90^o)$$

Problem 5:



(a) The impedance seen by the voltage source (also called the driving point impedance) is

$$Z(j\omega) = \frac{\frac{1}{j\omega C}(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} = \frac{R + j\omega L}{1 + rj\omega C + (j\omega)^2 LC}$$

To find the resonance frequency, we need to set the imaginary part of this to zero, and solve for ω.
(b) Since the voltage is $1\angle 0$, the current through the inductor is

$$i_L(t) = \frac{1\angle 0}{6 + j \times 3 \times 2} = \frac{1\angle 0}{\sqrt{72}\angle 45} = \frac{1}{\sqrt{72}}\angle - 45$$

The energy in the inductor depends on the current through the inductor as follows:

$$w_L(t) = \frac{1}{2}Li_L^2(t) = \frac{1}{2} \times 2 \times \frac{1}{72}\cos^2(3t - 45) = \frac{1}{72}\cos^2(3t - 45)$$

(c) The energy stored in the capacitor depends on the voltage across it, which is just $V_1(t)$.

$$w_C(t) = \frac{1}{2}CV_1^2(t) = \frac{1}{2} \times \frac{1}{36}\cos^2(3t) = \frac{1}{72}\cos^2(3t)$$

(d) The average power dissipated by the resistor is

$$P = \frac{1}{2}RI_{max}^2 = \frac{1}{2} \times 6 \times \frac{1}{72} = \frac{1}{24}$$

So the energy dissipated in one cycle is

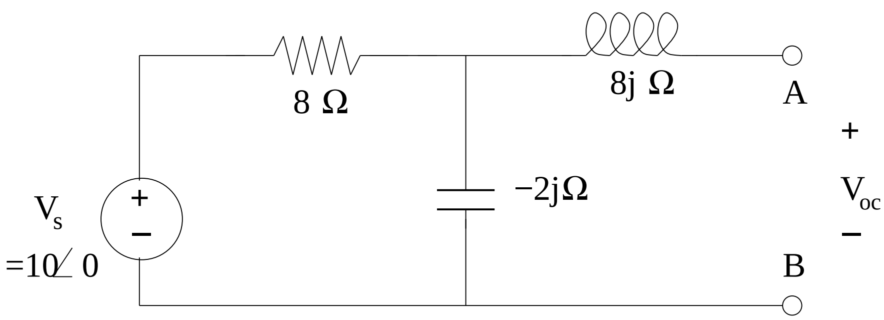
$$E = TP = \frac{2\pi}{3} \times \frac{1}{24} = \frac{\pi}{36}$$

(e) The quality factor is

$$Q = 2\pi \times \frac{\text{Max energy stored}}{\text{Energy diss. in 1 cycle}} = \frac{2\pi\frac{1}{72}[\cos^2(3t-45)+\cos^2(3t)]_{max}}{\frac{\pi}{36}} = [\cos^2(3t-45)+\cos^2(3t)]_{max}$$

and you can get full credit by just leaving the answer in this form.

Problem 6: (a) First we translate the circuit into the frequency domain:



Now V_{oc} can be found by voltage division

$$V_{oc} = \frac{-2j}{8-2j} \times 10\angle 0 = \frac{20\angle -90}{8.25\angle -14} = 2.42\angle -76$$

The Thevenin impedance is

$$Z_{th} = 8j + \frac{(-2j)(8)}{-2j+8} = \frac{16\angle -90}{8.25\angle -14} + 8j = 1/94\angle -76 + 8j = 0.47 - 1.88j + 8j = 0.47 + 6.12j$$

(b) The load impedance Z_L that absorbs the maximum average power is

$$Z_L = Z_{th}^* = 0.47 - 6.12j$$

(c) The current which flows is

$$I_L = \frac{V_{oc}}{Z_{th} + Z_L} = \frac{2.42\angle - 76}{0.47 + 6.12j + 0.47 - 6.12j} \frac{2.42\angle - 76}{0.94} = 2.57\angle - 76$$

The power absorbed is therefore

$$P_L = \frac{1}{2} R_L |I_L|^2 = \frac{1}{2} \times 0.47 \times 2.57^2 = 1.55W$$

(d) The load resistor R_L which would absorb the maximum power for resistive loads is

$$R_L = |Z_{th}| = \sqrt{0.47^2 + 6.12^2} = 6.13\Omega$$

Problem 7: If you understand what these plots of the transfer function are, then this problem takes only one minute. The input to the circuit is
$V_s(t) = 10 + 3\cos(5t) + 7\sin(10t) + 4\cos(15t + 30^o)$
From the plots:

$$H(j0) = 2\angle 0^o \qquad H(j5) = 0\angle - 45^o \qquad H(j10) = 5\angle - 90^o \qquad H(j15) = 1\angle - 45^o$$

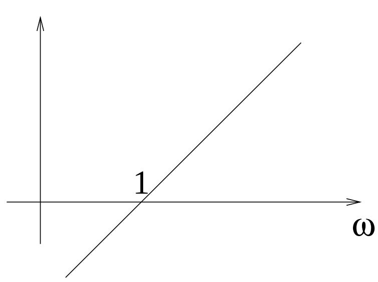
so the output is:

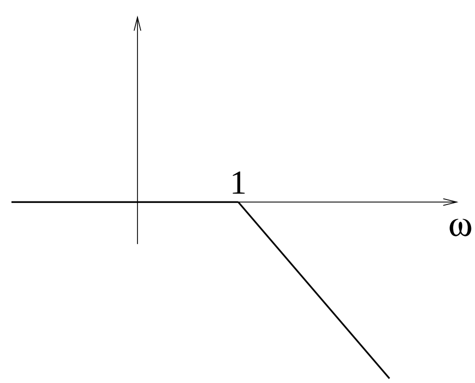
$$V_0(t) = 20 + 35\sin(10t - 90^o) + 4\cos(15t - 15^o)$$

Problem 8: The expression factors as:

$$H(j\omega) = (j\omega) \times \left(\frac{1}{1 + j\omega}\right) \times \left(\frac{1}{10 + j\omega}\right) = H_1(j\omega)H_2(j\omega)H_3(j\omega)$$

The first factor just goes up at 20 dB per decade, and crosses the axis at $\omega = 1$.
The second factor is a lowpass filter: the asymptote is flat to the left of $\omega = 1$ and goes down at 20dB per decade thereafter. These two factors look like this:





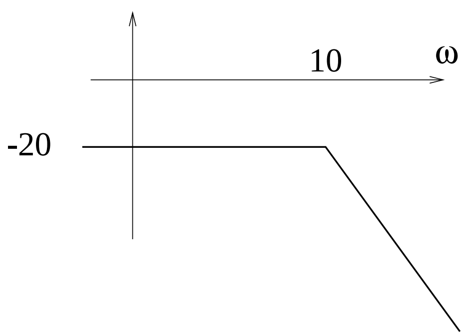
The third factor we will rewrite as

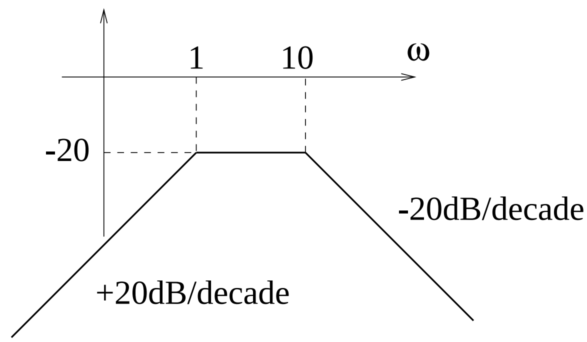
$$H_3(j\omega) = \frac{1}{10 + j\omega} = \frac{1/10}{1 + j\omega/10}$$

The denominator is now in the standard format from which we can say it corresponds also to a lowpass filter: the asymptote is flat to the left of $\omega = 10$. The numerator provides a constant offset of

$$20\log_{10} 1/10 = -20dB$$

The final Bode plot will just be the sum of the 3 plots. The H_3 factor and the final plot look like:





(b) This is a bandpass filter.

Problem 9:(a) The voltage transfer function is found by voltage division:

$$\begin{aligned}
H(j\omega) &= \frac{V_o}{V_{in}} = \frac{R}{R + j\omega L||(1/(j\omega C))} \\
&= \frac{R}{R + \frac{j\omega L \times 1/j\omega C}{j\omega L + 1/j\omega C}} \\
&= \frac{R(j\omega L + \frac{1}{j\omega C})}{R(j\omega L + \frac{1}{j\omega C}) + \frac{L}{C}} \\
&= \frac{RLC(j\omega)^2 + R}{RLC(j\omega)^2 + R + Lj\omega} \\
&= \frac{(j\omega)^2 + 1/LC}{(j\omega)^2 + 1/LC + j\omega/RC}
\end{aligned}$$

(b) If we look at $\omega \to 0$ and $\omega \to \infty$ for both of these cases, the magnitude of the transfer function goes to 1. At the resonance frequency $\omega_o = 1/\sqrt{LC}$, the numerator of the transfer function is zero, and the denominator of the transfer function is not zero there. So this is a notch filter (also called a bandstop filter or band rejection filter).

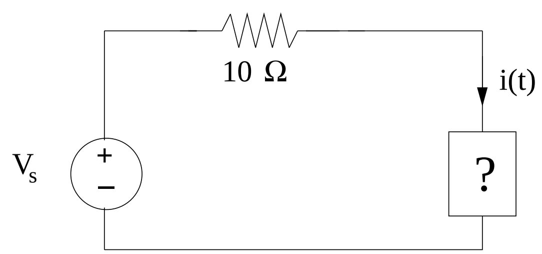
(c) To remove a 60-Hz hum, we need the transfer function to be zero at 60 Hz. So

$$\frac{1}{\sqrt{(LC)}} = 60 \times 2\pi$$

We are told C=10μF, so we just have to solve for L. The resistor value R does not come into it.

$$L = \frac{1}{C}\left(\frac{1}{60 \times 2\pi}\right)^2 = \frac{10^5}{377^2} = 0.704H$$

Problem 10: (a) Since $V_s(t) = 120\sqrt{2}\cos(120\pi t)$, phasor $V_s = 120\sqrt{2}\angle 0^o$



If we had an inductor in the box, then we would have

$$V_s = (R + j\omega L)I$$

which would mean that whatever angle I has, the angle of V_s is somewhat larger, since the $(R+j\omega L)$ term would be contributing a positive additional amount to the angle.
So, we need to have a capacitor in the box, so that the angle offset is the other way.
(b) To determine the numerical value:

$$I = \frac{V_s}{R - j/(\omega C)} = \frac{120\sqrt{2}\angle 0^o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}\angle \tan^{-1}\left(\frac{-1}{rC\omega}\right)}$$

Plugging in $R = 10$, we need:

$$\tan^{-1}\left(\frac{1}{10 \times 120\pi C}\right) = 55^o$$

$$C = \frac{1}{10 \times 120\pi \tan 55^o} = 186\mu F$$