MODELLING OF LIGHTNING DISCHARGE PATTERNS AS OBSERVED FROM SPACE

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ABSTRACT. Thunderstorms are an integral component in the Earth's atmospheric system and have profound influence on a variety of industries. Specifically, the occurrence of lightning discharges has many negative effects, including disturbances on power lines, the commencement of forest fires and the delay of aircraft missions. Consequently, modelling the patterns of lightning discharges is intended to provide insight into the thunderstorm processes, and may lead to their improved prediction.

This paper examines the current progress in the development of lightning models, both at the microphysical and abstracted numerical levels. In addition, a new mathematical model using percolation theory to represent thunderstorm images which were obtained through space shuttle videos is derived and evaluated.
KEYWORDS: current lightning modelling techniques, percolation theory, multifractals

1 INTRODUCTION

Thunderstorms are an essential component in the Earth's atmospheric system. One of the main phenomena associated with thunderstorms is lightning, which has many negative effects on a multitude of industries. For example, lightning is responsible for disturbances on power lines, the commencement of forest fires and the delay of aircraft missions. As a result, various lightning models have been developed to provide insight into the thunderstorm process and assist in the creation of better prediction techniques for thunderstorm displacement patterns.

This paper discusses some of the current models which have been developed to simulate the lightning discharge process, including their benefits and limitations. It will also present the concept and design for a new percolation-based model and discuss the preliminary results.

2 BACKGROUND

2.1 Data Acquisition

A variety of techniques are available to acquire actual lightning discharge data to be modelled, including both (i) ground-based techniques, and (ii) space-based programs. Ground-based systems are utilised to detect and record characteristics of *cloud-to-ground* (CG) lightning discharges during thunderstorms. The two main North American networks are the *National Lightning Detection Network* (NLDN) and the *Lightning Detection and Ranging* (LDAR) network. The NLDN is a system composed of magnetic direction finders spread across the United States and Canada, whereas the LDAR network is a specialised system employed by NASA at the Kennedy Space Center for assistance in space shuttle missions.

Lightning detection from space is currently performed by three main systems, with plans for an additional satellite underway, which are capable of detecting all types of lighting, including CG lightning, *inter-cloud* (IR) lightning and *intra-cloud* (IA) lightning. The *Optical Transient Detector* (OTD) and the *Lightning Imaging Sensor* (LIS) are two geosynchronous satellites which detect lightning discharges through the use of image analysis. The *Mesoscale Lightning Experiment* (MLE) is an ongoing project in which overhead videos of thunderstorms are recorded during space shuttle missions using a low level television camera. The proposed *Lightning Mapping Sensor* (LMS) would be a geostationary satellite which would permit the mapping of lightning discharges for a fixed area of Earth's surface.

2.2 Current Modelling Methods

A number of lightning discharge models have been developed so that better thunderstorm prediction techniques may be discovered. The majority of the current models attempt to simulate the actual physical processes and particles involved in a thunderstorm, typically at the microphysical level. This paper will focus on only two current models, namely the *Axisymmetric Numerical Cloud Model* (ANCM) [YaLT95] and the *3-Dimensional Unsymmetric Electrical Model* (3DUEM) [HaNK89].

The ANCM is an electrification model which represents a thundercloud in cylindrical, axisymmetric form. The authors selected the commonly used noninductive mechanism to model charge separation. An equation for the temporal and spatial space-charge distribution is then developed by solving a prognostic equation for the electrical space charge. The electrical potential is then calculated according to Poisson's equation. Finally, a *two dimensional* (2D) electric field vector may then be calculated from the potential using Gauss's law. A lightning discharge is said to occur when the electric field value at a given point exceeds a specified breakdown value. Charge redistribution after a discharge is modelled by neutralising the electric charge in a vertical column centered about the point of the lightning discharge.

The 3DUEM is a higher level electrification model which describes the evolution of the electric field within a thunderstorm. The output of this model is the electric field as a function of time while the inputs are the electrical currents generated by the flow of charged particles. A relationship between the input and the output is developed through the use of Maxwell's equations in the form of difference equations. Again, a lightning discharge is said to occur when the electric field exceeds a specified breakdown value. The lightning discharge itself is modelled by breaking the time step in which it occurs into smaller segments and using the inverse matrix modification formula.

Although the majority of the current models have the benefit of simulating realistic physical properties, this methodology is responsible for one of the major limitations of these models. Due to the small size of the particles involved in the thunderstorm process as compared with the actual thunderstorm itself, current models tend to be quite large and complicated. Another limitation of the current models is the lack of representation of the chaotic nature displayed by the lightning discharge patterns. Consequently, a new model is required. An ideal candidate for the basis of this model is percolation theory, due to the similarity between percolation and lightning images. A Laplacian multifractal model of dielectric discharges has been studied extensively [Stac94][Aren96].

2.3 Percolation Theory

Percolation is a multifractal model which can represent simply a disordered system [Vics92][Fede88]. The most basic form of percolation is site percolation. Consider a square lattice where each square is either empty or filled. A number of seeds are placed within the lattice, marking their squares as filled. The number and location of these seeds may be fixed or random. For a specified length of time, consider each of the nearest neighbour squares of all the filled squares and generate a random number between 0 and 1 for each. If this number is greater than a predetermined spreading probability, p, then the square is marked as filled. Otherwise, the square is marked as dead for the rest of the simulation. The percolation process is quite sensitive for a small range of p and will create fractal structures for these values. When the value of p deviates too greatly from this range, the percolation is generally uninteresting, as the majority of the lattice is either filled or empty.

A simple example of 2D percolation is shown in Fig. 1. The five grids represent five successive time steps in the growth of the percolation fractal. In the first step, random probabilities are generated for each of the squares in the five by five grid and an initial seed is placed. Using a spreading probability of 0.5, the percolation fractal spreads outward as shown in the subsequent three grids.

0.95	0.93	0.04	0.11	0.70
0.96	0.63	0.81	0.09	0.99
0.20	0.40		0.12	0.24
0.85	0.37	0.60	0.61	0.08
0.02	0.01	0.70	0.70	0.88

0.95	0.93	0.04	0.11	0.70
0.96	0.63		0.09	0.99
0.20	0.40		0.12	0.24
0.85	0.37		0.61	0.08
0.02	0.01	0.70	0.70	0.88

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0.20	0.40		0.12	0.24
0.85	0.37			0.08
0.02	0.01		0.70	0.88

0.25		0.04	0.11	0.70
			0.09	0.99
0.20	0.40		0.12	0.24
0.85	0.37			0.08
0.02	0.01			0.48

Fig. 1. Four time steps in the growth of a 2D percolation fractal.

A number of variations on the percolation model exist. In addition to slight modifications to the site percolation algorithm, bond percolation and continuum percolation are two other types of percolation. As well, although the percolation model is most easily presented in 2D, it may be slightly modified for three or higher dimensions. For *three dimensions* (3D), the only required change is the inclusion of the upper and lower nearest neighbour sites when considering the squares around a filled site.

3 PERCOLATION MODEL OF LIGHTNING

The model of lightning discharge patterns presented in the paper is one which is based on percolation theory. The source of real lightning data for this model is space shuttle video recorded as part of the MLE and shown in Fig. 2. One may clearly see the locations of the lightning discharges in the thunderstorm, allowing the greyscale images of the video to be easily converted to black and white images, as shown in Fig. 3.

Depending on the degree of sophistication required by the percolation model, the simulation may either be performed in 2D or 3D. In 2D, a lightning discharge is said to occur when a branch of the percolation fractal intersects the boundary of the lattice. In 3D, a lightning discharge is said to occur when the percolation fractal reaches the upper-most plane of the lattice. In effect, any given plane will very closely model the 2D images as seen from space.

Regardless of the embedding dimension, the frames for the lightning discharge videos may be generated in one of two ways. Most simplistically, for each frame, new seeds are placed and the percolation occurs, effectively modelling only the birth of new lightning discharges. Alternately, the

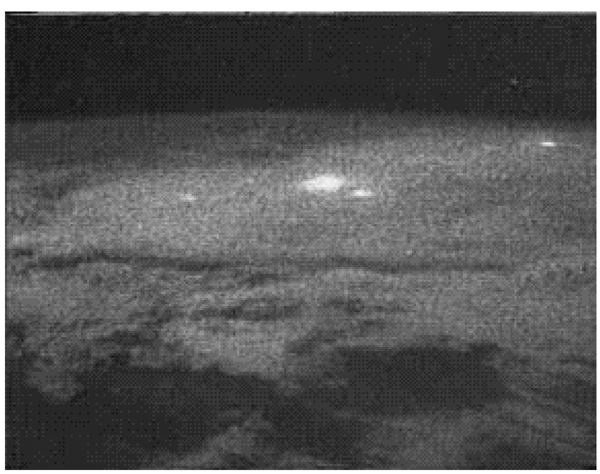

Fig. 2. Single frame from the shuttle lightning video.
[Courtesy of the Engineering Photo Analysis Group of the MSFC propulsion laboratory.]

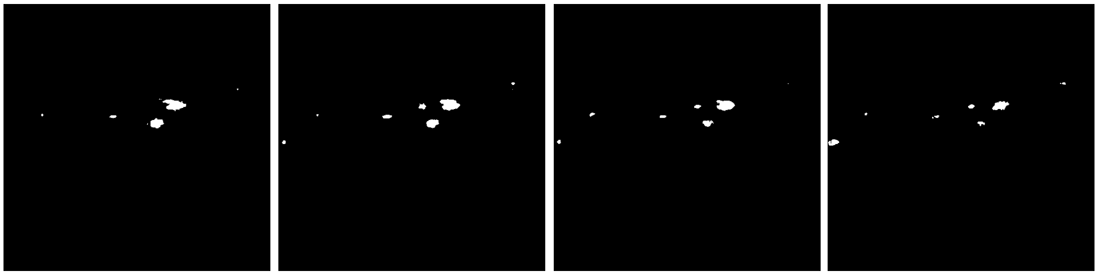

Fig. 3. Successive frames from the black and white representation of the shuttle lightning video.

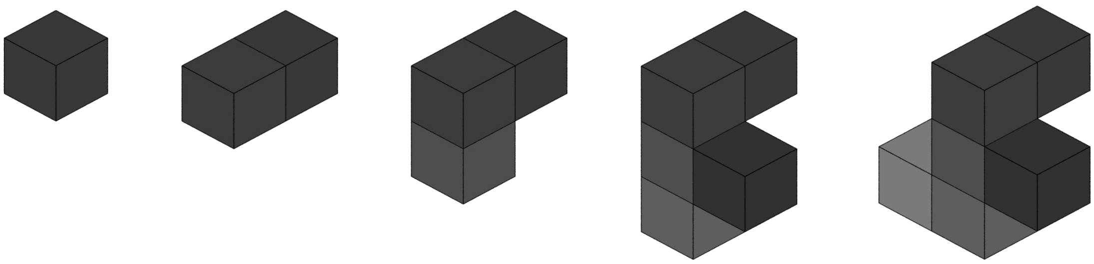

Fig. 4. Five time steps in the growth of a 3D percolation fractal.

correlation of lightning locations between frames may be included in the percolation model by generating successive frames of the video from successive layers in the 3D lattice.

A simple example of 3D percolation using the correlation method with a lattice size of five and a single seed is displayed in Fig. 4. The five images represent the growth the of fractal through five time steps. In each step, the addition of a new filled nearest neighbour site indicates that the random probability generated for this square was greater than the spreading probability. Successive frames for the lightning video are then created from the successive vertical layers of the final percolation fractal.

4 EXPERIMENTAL RESULTS AND DISCUSSION

In this research, a 3D lattice is used for the percolation model and successive frames are generated using the correlation method. The lattice employed has dimensions of 128 by 128 by 900 and each frame is generated by scaling each side of a 128 by 128 layer by a factor of four to generate 512 by 512 images. Four hundred seeds are randomly distributed throughout the lattice. The percolation is responsible for the determination of the locations of the lightning discharges as well as their movement and growth with time. By varying the value of the parameter p, the spreading probability, three major classes of percolation fractals may be developed. When the p value is low the lightning discharges occupy a large percentage of the thunderstorm, whereas when the p value is high the lightning discharges occupy almost none of the thunderstorm. Both these cases are not commonly observed in real data, however, they demonstrate the capabilities of the percolation model. When p values in the range of 0.70 to 0.72 are considered, the resulting videos more closely resemble those observed from space. Figures 5 through 8 show four successive frames from 300 frame videos generated using p values of 0.69, 0.75, 0.70 and 0.72, respectively.

To examine the results of the percolation model, an analysis is performed on the video created from the model using $p = 0.70$. The simplest means of evaluating the model is a visual inspection. As can be seen from the selected frames, and more clearly from the video itself, the percolation-based movies appear to model the lightning discharge patterns in the shuttle video quite closely. However, a more precise method of assessment is required for an accurate comparison. Due to the nonstationarity of the lightning discharges, statistical methods are inadequate, and thus fractal analysis should be performed.

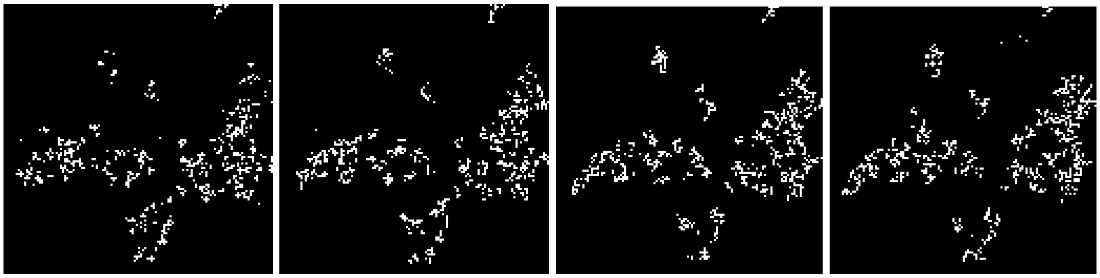

Fig. 5. Successive frames from the percolation video with $p = 0.69$.

Fig. 6. Successive frames from the percolation video with $p = 0.75$.

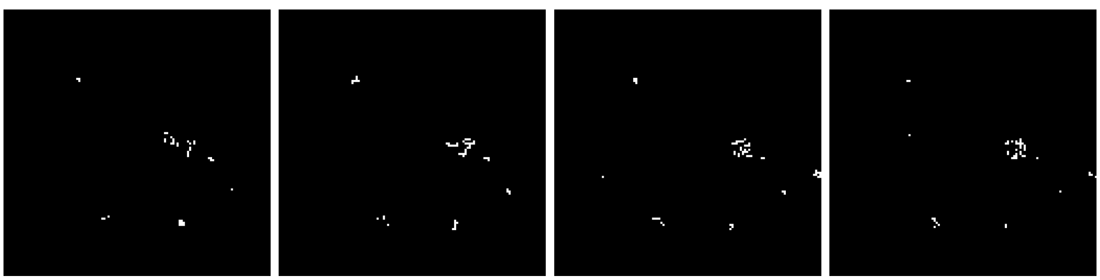

Fig. 7. Successive frames from the percolation video with $p = 0.72$.

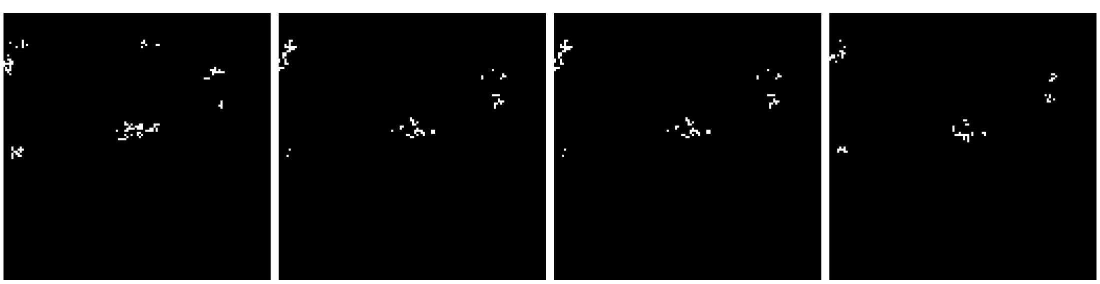

Fig. 8. Successive frames from the percolation video with $p = 0.70$.

To compare the model with the real data, the Rényi dimension [Kins94] of the differences between successive frames, a fractal structure, is calculated for each of the two movies. To calculate this multifractal spectrum, we cover the image with 2D *volumetric elements* (vels) with a fixed radius, r. We then repeat this covering, allowing the radius of the vels to go to 0. From this we may calculate the probability, p_j, of the jth vel intersecting the fractal and also N_r, the summation over all vels of the number of intersections with the fractal for the current r. The Rényi dimension is based on Rényi's generalisation of entropy with respect to the moment order, $-\infty \le q \le \infty$. Assuming a power-law relation we conclude that the Rényi dimension is given by

$$D_q = \lim_{r \to 0} \frac{1}{1-q} \frac{\log \sum_{j=1}^{N_r} \left(p_j^q\right)}{\log(1/r)} \tag{1}$$

The Rényi spectrum, which is a nonincreasing function, is most easily viewed in a D_q vs q vs frame plot. The plot of the Rényi spectrum for the shuttle video is shown in Fig. 9 while that for the percolation video is shown in Fig. 10. These two plots are clearly quite similar in overall structure. In both cases, the D_q values range from approximately 1.0 to 2.0 and the curve drops quite rapidly around the value of $q = 0$. This drastic change in the D_q values with q illustrates that the lightning discharge patterns are spatially multifractal. There exist a small number of exceptions to this general shape in the spectrum of the actual lightning video, where D_q remains constant at a value of 2. This anomaly indicates that the image does not contain any filled pixels, which is caused by the lack of change between two successive video frames. The major differences between the two plots occur with respect to the frame number. By considering the D_q values for negative q, it may be seen that the actual lightning video is quite constant with a value of 2, while D_q for the percolation video ranges between approximately 2.2 and 2. Conversely, for positive q values, the actual video possesses a D_q range from approximately 0.8 to 1.2 while that of the percolation video is 1 to 1.4. Variation in D_q for either low or high q values indicates the non-stationarity of the discharge patterns. The occurrence of these ripples demonstrates that the actual video is sensitive predominantly to higher p_j values (positive q values) whereas the percolation video is sensitive to both high and low p_j values. Another noteworthy aspect of the percolation spectrum is the presence of ripples which are nearly regular, indicating that the growth of the discharges is slightly more oscillatory. The changes in D_q with respect to frame number imply that lightning discharge patterns are also multifractals in time. Despite the slight differences between the two Rényi spectrums, their general similarity demonstrates that the percolation model creates videos which are accurate representations of lightning videos as observed from the shuttle.

5 CONCLUSIONS

The severe impact and constant presence of lightning discharges in thunderstorms has motivated the generation of a variety of models capable of simulating these patterns. This paper discussed two such current models, namely the Axisymmetric Numerical Cloud Model [YaLT95] and the 3-Dimensional Unsymmetric Electrical Model [HaNK89]. As illustrated in these two cases, many current methods are overly complex, due to an attempt to model the actual physical processes within a thunderstorm, normally at the microphysical level. Hence, a simple percolation-based Vicsek [Vics92] and Feder [Fede88] model was developed to simulate discharge patterns as seen in videos captured by a space shuttle. A representative movie was created through percolation and evaluated using the Rényi dimension. This analysis suggests that lightning discharge patterns are multifractal in both space and time, and that percolation theory may be easily employed to create lightning models for better understanding and prediction of the thunderstorm process.

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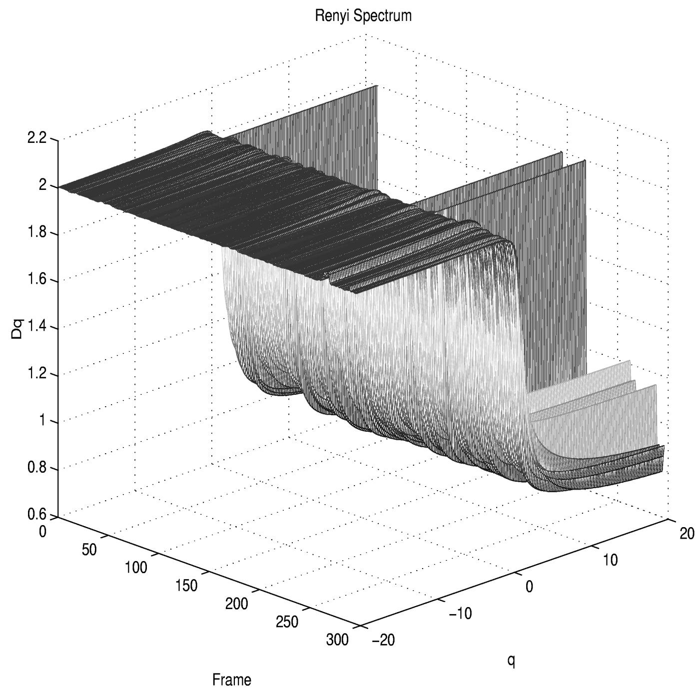

Fig. 9. Rényi spectrum for the black and white representation of the shuttle lightning video.

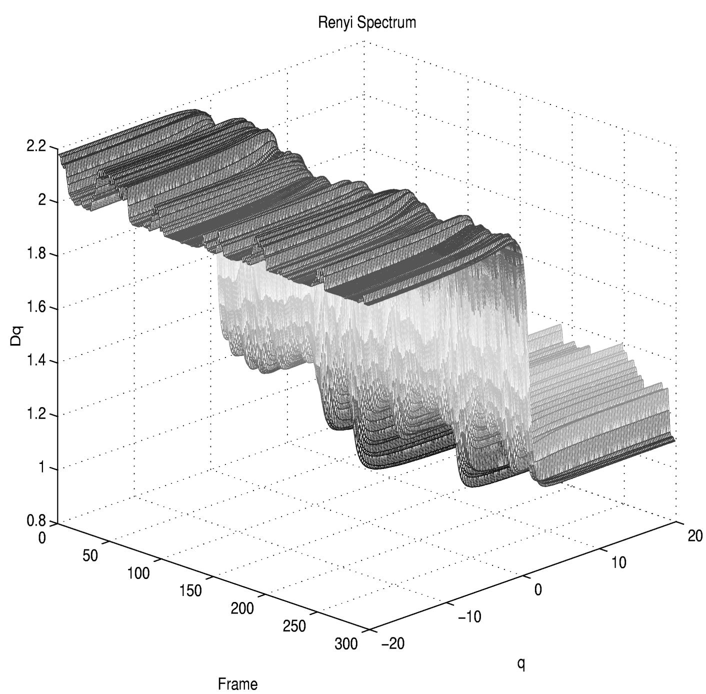

Fig. 10. Rényi spectrum for the percolation video with p = 0.70.

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