Wireless Relay Placement

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***Abstract*— An algorithm is given for placing relays at spatial positions to improve the reliability of communicated data in a sensor network. The network consists of many power-limited sensors, a small set of relays, and a receiver. The receiver receives a signal directly from each sensor and also indirectly via a single-hop relay path. The relays rebroadcast the transmissions in order to achieve diversity at the receiver. Both amplify-and-forward and decode-and-forward relay networks are considered. Channels are modeled with Rayleigh fading, path loss, and additive white Gaussian noise. The main results of the paper are geometric descriptions of sets of locations in the plane in which sensors are assigned to given, fixed-location relays, and the analysis of system performance.**

I. Introduction

Wireless sensor networks typically consist of a large number of small, power-limited sensors distributed over a planar geographic area. In some scenarios, the sensors collect information which is transmitted to a single receiver for further analysis. A small number of radio relays with additional processing and communication capabilities can be strategically placed to help improve system performance. Two important problems we consider here are to position the relays and to determine, for each sensor, which relay should rebroadcast its signal.

Previous studies of relay placement have considered various optimization criteria and communication models. For example, coverage, lifetime, energy usage, error probability, outage probability, or throughput were focused on by [1], [3]–[5], [7], [9]–[18]. The communication and/or network models used are typically simplified by making assumptions such as: error-free communications, transmission energy is an increasing function of distance, single-sensor networks, single-relay networks, and no diversity.

In this work, we attempt to minimize the average probability of error, and use a more elaborate communications model which includes path loss, fading, additive white Gaussian noise, and diversity. We present an algorithm that determines relay placement and, for each sensor, which relay should rebroadcast its transmissions. We refer to this algorithm as the *relay placement algorithm*. We also describe geometrically, with respect to fixed relay positions, the sets of locations in the plane in which sensors are (optimally) assigned to the same relay.

II. Communications Model and Performance Measure

A. Signal, Channel, and Receiver Models

In a sensor network, we refer to sensors, relays, and the receiver as *nodes*. We assume that transmission of $b_i \in \{-1, 1\}$ by node i uses the binary phase shift keyed (BPSK) signal $s_i(t)$, and we denote the transmission energy per bit by E_i. In particular, we assume all sensor nodes transmit at the same energy per bit, denoted by E_{Tx}. We assume TDMA communications by sensors and relays so that there is (ideally) no transmission interference. Let $L_{i,j}$ denote the far field path loss between two nodes i and j that are separated by a distance $d_{i,j}$ (in meters). We consider the free-space law model for path loss for which

$$L_{i,j} = \frac{F_2}{d_{i,j}^2} \tag{1}$$

where:

$F_2 = \frac{\lambda^2}{16\pi^2}$ (in meters^2)
$\lambda = c/f_0$ is the carrier wave wavelength (in meters)
$c = 3 \cdot 10^8$ is the speed of light (in meters/second)
f_0 is the frequency of the carrier wave (in Hz).

The formula in (1) is impractical in the near field, since $L_{i,j} \to \infty$ as $d_{i,j} \to 0$. Comaniciu and Poor [6] addressed this issue by not allowing transmissions at distances less than λ. Ong and Motani [13] allow near field transmissions by proposing a modified model with path loss

$$L_{i,j} = \frac{F_2}{(1 + d_{i,j})^2}. \tag{2}$$

We assume additive white Gaussian noise $n_j(t)$ at the receiving antenna of node j. The noise has one-sided power spectral density N_0 (in W/Hz). Assume the channel fading (excluding path loss) between nodes i and j is a random variable $h_{i,j}$ with Rayleigh density $p_{h_{i,j}}(h) = (h/\sigma^2)e^{-h^2/(2\sigma^2)}$, with $h \geq 0$.

Let the signal received after transmission from node i to node j be denoted by $r_{i,j}(t) = \sqrt{L_{i,j}}\, h_{i,j} s_i(t) + n_j(t)$. The average (over the fade) received energy per bit is $E_j = 4\sigma^2 E_i L_{i,j}$. We assume demodulation at a receiving node is performed by applying a matched filter to obtain the test statistic. Diversity is achieved at the receiver by selection combining, in which only the better of the two incoming signals (determined by a measurable quantity such as the received signal-to-noise-ratio (SNR)) is used to detect the transmitted bit.

B. Path Probability of Error

For each sensor, we determine the probability of error along the direct path from the sensor to the receiver and along single-hop[1] relay paths, for both amplify-and-forward and decode-and-forward protocols. Each transmitter will be denoted by a position $\mathbf{x} \in \mathbb{R}^2$, and the receiver is denoted by Rx. We consider transmission paths of the forms $(\mathbf{x}, \mathrm{Rx})$, $(\mathbf{x}, i)$, (i, Rx), and $(\mathbf{x}, i, \mathrm{Rx})$, where i denotes a relay index. For each such path q, let:

$$\begin{aligned} \mathrm{SNR}_H^q &= \text{end-to-end SNR, conditioned on the fades} \\ P_{e|H}^q &= \text{end-to-end error probability,} \\ &\quad\ \text{conditioned on the fades} \\ \mathrm{SNR}^q &= \text{end-to-end SNR, averaged over the fades} \\ P_e^q &= \text{end-to-end error probability,} \\ &\quad\ \text{averaged over the fades.} \end{aligned}$$

Note that the signal-to-noise ratios only apply to direct paths and paths using amplify-and-forward relays. Finally, denote the Gaussian error function by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$.

1) Direct Path (i.e., unrelayed): For Rayleigh fading, it can be shown that, for each transmitter $\mathbf{x}$,

$$\begin{aligned} \mathrm{SNR}^{(\mathbf{x},\mathrm{Rx})} &= \frac{4\sigma^2 E_{\mathrm{Tx}} L_{\mathbf{x},\mathrm{Rx}}}{N_0} \\ \mathrm{SNR}^{(\mathbf{x},i)} &= \frac{4\sigma^2 E_{\mathrm{Tx}} L_{\mathbf{x},i}}{N_0} \\ \mathrm{SNR}^{(i,\mathrm{Rx})} &= \frac{4\sigma^2 E_i L_{i,\mathrm{Rx}}}{N_0} \end{aligned}$$

$$P_e^{(\mathbf{x},\mathrm{Rx})} = \frac{1}{2}\left(1 - \left(1 + \frac{2}{\mathrm{SNR}^{(\mathbf{x},\mathrm{Rx})}}\right)^{-1/2}\right). \quad (4)$$

Note that analogous formulas to that in (4) can be given for $P_e^{(\mathbf{x},i)}$ and $P_e^{(i,\mathrm{Rx})}$.

2) Relay Path with Amplify-and-Forward: For amplify-and-forward, the system is linear. Denote the gain by G. Conditioning on the fading values, we have (e.g., see [8])

$$\begin{aligned} \mathrm{SNR}_H^{(\mathbf{x},i,\mathrm{Rx})} &= \frac{h_{\mathbf{x},i}^2 h_{i,\mathrm{Rx}}^2 E_{\mathrm{Tx}}/N_0}{B_i h_{i,\mathrm{Rx}}^2 + D_i} \\ P_{e|H}^{(\mathbf{x},i,\mathrm{Rx})} &= Q\left(\sqrt{\mathrm{SNR}_H^{(\mathbf{x},i,\mathrm{Rx})}}\right) \end{aligned}$$

$$\text{where} \quad B_i = \frac{1}{2L_{\mathbf{x},i}}; \qquad D_i = \frac{1}{2G^2 L_{\mathbf{x},i} L_{i,\mathrm{Rx}}}.$$

[1]Computing the probabilities of error for the more general case of multi-hop relay paths is straightforward.

Then, the end-to-end probability of error, averaged over the fades, can be shown to be

$$\begin{aligned} P_e^{(\mathbf{x},i,\mathrm{Rx})} = \frac{1}{2} - &\frac{D_i \sqrt{\pi} N_0/E_{\mathrm{Tx}}}{8\sigma\left(\sigma^2 + B_i N_0/E_{\mathrm{Tx}}\right)^{3/2}} \\ &\cdot U\left(\frac{3}{2}, 2, \frac{D_i N_0/E_{\mathrm{Tx}}}{2\sigma^2\left(\sigma^2 + B_i N_0/E_{\mathrm{Tx}}\right)}\right) \end{aligned}$$

where $U(a,b,z)$ denotes the confluent hypergeometric function of the second kind, i.e.,

$$U(a,b,z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1}\, dt.$$

3) Relay Path with Decode-and-Forward: For decode-and-forward relays, the signal at the receiver is not a linear function of the transmitted signal, as the relay makes a hard decision based on its incoming data. A decoding error occurs at the receiver if and only if exactly one decoding error is made along the relay path. Thus, for Rayleigh fading, we have (e.g., see [8])

$$\begin{aligned} P_e^{(\mathbf{x},i,\mathrm{Rx})} = \frac{1}{4}&\left(1 - \left(1 + \frac{2}{\mathrm{SNR}^{(\mathbf{x},i)}}\right)^{-1/2}\right) \\ &\cdot \left(1 + \left(1 + \frac{2}{\mathrm{SNR}^{(i,\mathrm{Rx})}}\right)^{-1/2}\right) \\ + \frac{1}{4}&\left(1 - \left(1 + \frac{2}{\mathrm{SNR}^{(i,\mathrm{Rx})}}\right)^{-1/2}\right) \\ &\cdot \left(1 + \left(1 + \frac{2}{\mathrm{SNR}^{(\mathbf{x},i)}}\right)^{-1/2}\right). \end{aligned}$$

III. Path Selection and Relay Placement Algorithm

A. Definitions

We define a *sensor network with relays* to be a collection of sensors and relays in $\mathbb{R}^2$, together with a single receiver at the origin, where each sensor transmits to the receiver both directly and through some predesignated relay for the sensor, and the system performance is evaluated using the measure given below in (5). Specifically, let $\mathbf{x}_1, \ldots, \mathbf{x}_M \in \mathbb{R}^2$ be the sensor positions and let $\mathbf{y}_1, \ldots, \mathbf{y}_N \in \mathbb{R}^2$ be the relay positions. Typically, $N \ll M$. We call any map $p : \mathbb{R}^2 \to \{1, \ldots, N\}$ a *sensor-relay assignment*, where $p(\mathbf{x}) = i$ means that if a sensor were located at position $\mathbf{x}$, then it would be assigned to relay $\mathbf{y}_i$ (i.e., its transmission would be rebroadcast by $\mathbf{y}_i$). Let $\mathcal{S}$ be a bounded subset of $\mathbb{R}^2$. Henceforth, we will only consider sensor-relay assignments whose domains are restricted to $\mathcal{S}$ (since the number of sensors is finite). Let the *sensor-averaged probability of error* be given by

$$\frac{1}{M} \sum_{s=1}^{M} P_e^{(\mathbf{x}_s, p(\mathbf{x}_s), \mathrm{Rx})}. \quad (5)$$

Note that (5) depends on the relay locations through the sensor-relay assignment p.

B. Overview of the Proposed Algorithm

The proposed iterative algorithm attempts to minimize the sensor-averaged probability of error[2] over all choices of relay positions $\mathbf{y}_1, \ldots, \mathbf{y}_N$ and sensor-relay assignments p. The algorithm operates in two phases. First, the relay positions are fixed and the best sensor-relay assignment is determined; second, the sensor-relay assignment is fixed and the best relay positions are determined. An initial placement of the relays is made either randomly or using some heuristic. The two phases are repeated until the quantity in (5) has converged within some threshold.

C. Phase 1: Optimal Sensor-Relay Assignment

In the first phase, we assume the relay positions $\mathbf{y}_1, \ldots, \mathbf{y}_N$ are fixed and choose an optimal[3] sensor-relay assignment p^*, in the sense of minimizing (5). This choice can be made using an exhaustive search in which all possible sensor-relay assignments are examined. A sensor-relay assignment induces a partition of $\mathcal{S}$ into subsets for which all sensors in any such subset are assigned to the same relay. For each relay $\mathbf{y}_i$, let σ_i be the set of all points $\mathbf{x} \in \mathcal{S}$ such that if a sensor were located at position $\mathbf{x}$, then the optimally assigned relay that rebroadcasts its transmissions would be $\mathbf{y}_i$, i.e., $\sigma_i = \{\mathbf{x} \in \mathcal{S} : p^*(\mathbf{x}) = i\}$. We call σ_i the i^{th} *optimal sensor region* (with respect to the fixed relay positions).

D. Phase 2: Optimal Relay Placement

In the second phase, we assume the sensor-relay assignment is fixed and choose optimal[4] relay positions in the sense of minimizing (5). Numerical techniques can be used to determine such optimal relay positions. For the first three instances of phase 2 in the iterative algorithm we used an efficient (but slightly sub-optimal) numerical approach that quantizes a bounded subset of $\mathbb{R}^2$ into gridpoints. For a given relay, the best gridpoint was selected as the new location for the relay. For subsequent instances of phase 2, the restriction of lying on a gridpoint was removed and a steepest descent technique was used to refine the relay locations.

IV. Geometric Descriptions of Optimal Sensor Regions

We now geometrically describe each optimal sensor region by considering specific relay protocols and channel models.[5] In particular, we examine amplify-and-forward and decode-and-forward relaying protocols in conjunction with Rayleigh fading channels. We define the *internal boundary* of any optimal sensor region σ_i to be the portion of the boundary of σ_i that does not lie on the boundary of $\mathcal{S}$. We show that as the transmission energies of sensors and relays grow, the internal boundary of each optimal sensor region becomes piecewise linear.

For each pair of relays $(\mathbf{y}_i, \mathbf{y}_j)$, let $\sigma_{i,j}$ be the set of all points $\mathbf{x} \in \mathcal{S}$ such that if a sensor were located at position $\mathbf{x}$, then its average probability of error using relay $\mathbf{y}_i$ would be smaller than that using relay $\mathbf{y}_j$, i.e., $\sigma_{i,j} = \left\{\mathbf{x} \in \mathcal{S} : P_e^{(\mathbf{x},i,\mathrm{Rx})} < P_e^{(\mathbf{x},j,\mathrm{Rx})}\right\}$. Note that $\sigma_{i,j} = \mathcal{S} - \sigma_{j,i}$. Then, for the given set of relay positions, we have $\sigma_i = \bigcap_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{i,j}$ since $p^*(\mathbf{x}) = \underset{j \in \{1,\ldots,N\}}{\operatorname{argmin}} P_e^{(\mathbf{x},j,\mathrm{Rx})}$. Furthermore, in order to facilitate analysis, for a suitably chosen constant $C > 0$, we modify (2) to[6]

$$L_{i,j} = \frac{F_2}{C + d_{i,j}^2}. \tag{6}$$

Also, we define the *nearest-neighbor region* of a relay $\mathbf{y}_i$ to be $\{\mathbf{x} \in \mathcal{S} : \forall j, \ \|\mathbf{x} - \mathbf{y}_i\| < \|\mathbf{x} - \mathbf{y}_j\|\}$, where ties (i.e., $\|\mathbf{x} - \mathbf{y}_i\| = \|\mathbf{x} - \mathbf{y}_j\|$) are broken arbitrarily. The interiors of these regions are convex polygons intersected with $\mathcal{S}$.

Theorem 4.1 ([2]): Consider a sensor network with amplify-and-forward relays and Rayleigh fading channels, and let $E_{\mathrm{Tx}}/N_0 \to \infty$. Then, each optimal sensor region is asymptotically equal to the corresponding relay's nearest-neighbor region.

Theorem 4.2 ([2]): Consider a sensor network with decode-and-forward relays and Rayleigh fading channels, and, for all relays i, let $E_i/N_0 \to \infty$ and $E_{\mathrm{Tx}}/N_0 \to \infty$ such that $(E_i/N_0)/(E_{\mathrm{Tx}}/N_0)$ has a limit. Then, the internal boundary of each optimal sensor region is asymptotically piecewise linear.

V. Numerical Results

The relay placement algorithm was implemented for both amplify-and-forward and decode-and-forward relays. The sensors were placed uniformly in a square of side-length 100 m. For all decode-and-forward relays $\mathbf{y}_i$, each energy E_i was set to a constant, E. The value of E was selected so that the total transmission energy (summed over all relays) was the same for both amplify-and-forward and decode-and-forward. Specific numerical values for system variables were selected as follows: $f_0 = 900$ MHz, $\sigma = \sqrt{2}/2$, $M = 10000$, $C = 1$, $G = 75$ dB, $E_{\mathrm{Rx}}/N_0|_{d=50\text{ m}} = 8$ dB.

In order to use the relay placement algorithm to produce good relay locations and sensor-relay assignments, we ran the algorithm 10 times. Each such run was initiated

[2]Here we minimize (5); however, the algorithm can be adapted to minimize other performance measures.

[3]This choice may not be unique, but we select one such minimizing assignment. Also, optimality of p^* here depends only on the values $p^*(\mathbf{x}_1), \ldots, p^*(\mathbf{x}_M)$.

[4]This choice may not be unique, but we select one such set of positions.

[5]Additional derivations can be found in [2].

[6]Numerical results confirm that (6) is a close approximation of (2) for parameters of interest.

with a different random set of relay locations (uniformly distributed on the square $\mathcal{S}$) and used the sensor-averaged probability of error given in (5). For each of the 10 runs completed, 1000 simulations were performed with Rayleigh fading and diversity (selection combining) at the receiver. Different realizations of the fade values for the sensor network channels were chosen for each of the 1000 simulations. Of the 10 runs, the relay locations and sensor-relay assignments of the run with the lowest average probability of error over the 1000 simulations was chosen.

Figure 1 gives the output for $N = 12$ amplify-and-forward relays using exact probability of error expressions. Relays are denoted by black squares and the receiver is denoted by a black circle at the origin. Boundaries between the optimal sensor regions are shown in black. The sensor-averaged probability of error for the results in Figure 1 is 1.6×10^{-2}. The relay placements and sensor assignments are very similar for amplify-and-forward and decode-and-forward. The sensor error probability is lowest for sensors that are closest to the relays, and increases with distance. Since the relays transmit at higher energies than the sensors, the probability of detection error is reduced by reducing path loss before a relay rebroadcasts a sensor's signal, rather than after the relay rebroadcasts the signal (even at the expense of possibly greater path loss from the relay to the receiver). Thus, some sensors actually transmit "away" from the receiver to their associated relay.

The asymptotically-optimal sensor regions described in Theorems 4.1 and 4.2 very closely matched those corresponding to the exact probability of error expressions, particularly for decode-and-forward; for amplify-and-forward the visibly slightly-curved boundaries in the corner regions of $\mathcal{S}$ in Figure 1 were approximated with straight lines.

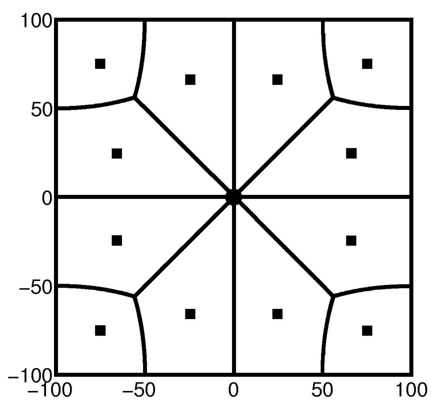

Fig. 1. Algorithm and simulation output for amplify-and-forward over fading channels with $N = 12$, $G = 75$ dB, and $E_{\mathrm{Rx}}/N_0|_{d=50\text{ m}} = 5$ dB. Relays are denoted by black squares and the receiver is located at $(0, 0)$. Sensors are distributed as a square grid over ± 100 meters in each dimension. The optimal sensor regions $\sigma_1, \ldots, \sigma_{12}$ are separated by black boundaries.

Acknowledgment

This work was supported by the Air Force Office of Scientific Research award FA9550-06-1-0210, the National Science Foundation, and the UCSD Center for Wireless Communications.

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