

Solutions for Homework 4

1. Chromaticity diagrams:

Here is the matlab code to plot the tristimulus values:

```
>> load cie -ascii
>> l = cie(:,1);
>> X = cie(:,2);
>> Y = cie(:,3);
>> Z = cie(:,4);
>> plot(l,X,'r-',l,Y,'g--',l,Z,'b-.');
```

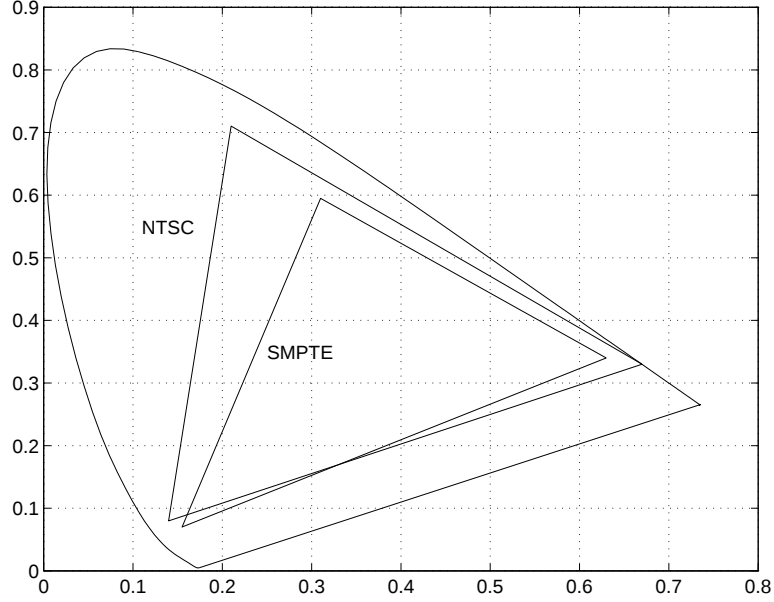
Here is the code to plot the chromaticity diagram. The connected line of purples is obtained by simply tacking the first point of each vector on to its end:

```
>> x = X ./ (X+Y+Z);
>> y = Y ./ (X+Y+Z);
>> x = [x;x(1)];
>> y = [y;y(1)];
>> plot(x,y)
```

You were given the conversions from XYZ to RGB. What we need, however, is to convert from RGB to XYZ in order to plot in the xy chromaticity diagram. Specifically, we need to convert each of the pure phosphor colors to XYZ, then to xy. For example, the vector $[1 \ 0 \ 0]'$ represents the pure NTSC red phosphor. That is, it is the NTSC red primary expressed in the NTSC coordinates. By using the inverse of the given matrix, we can convert this $(1 \ 0 \ 0)$ point from the NTSC space to the XYZ space. Likewise, we have to convert $(0 \ 1 \ 0)$ and $(0 \ 0 \ 1)$ to the XYZ space. These three XYZ tristimulus values then get converted to xy and plotted in the xy plane. Here is the code to plot the NTSC triangle inside the xy-chromaticity horse-shoe:

```
>> Nmat = [1.910 -0.532 -0.288; -0.985 2.0 -0.028; 0.058 -0.118 0.898];
>> invNmat = inv(Nmat);
>> xyzR = invNmat * [1 0 0]';
>> xR = xyzR(1) / sum(xyzR);
>> yR = xyzR(2) / sum(xyzR);
>> xyzG = invNmat * [0 1 0]';
>> xG = xyzG(1) / sum(xyzG);
>> yG = xyzG(2) / sum(xyzG);
>> xyzB = invNmat * [0 0 1]';
>> xB = xyzB(1) / sum(xyzB);
>> yB = xyzB(2) / sum(xyzB);
>> a = [xR xG xB xR];
>> b = [yR yG yB yR];
>> plot(x,y,a,b)
>> grid
```

We do the same thing to get the SMPTE phosphors. The NTSC set is bigger, although we have to remember that the space is not perceptually uniform, and so the amount of advantage can't be estimated from this diagram.



2. Adding Colors

(a) Tristimulus values are additive. The tristimulus values for W are (1,1,1). So if we use R,G,B to denote the tristimulus values for color A, we have

$$R = \frac{1}{2}(T_1 + 1) \quad \text{and} \quad G = \frac{1}{2}(T_2 + 1) \quad \text{and} \quad B = \frac{1}{2}(T_3 + 1)$$

The chromaticity coordinates are

$$r = \frac{R}{R + G + B} = \frac{T_1 + 1}{T_1 + T_2 + T_3 + 3} \quad \text{and} \quad g = \frac{G}{R + G + B} = \frac{T_2 + 1}{T_1 + T_2 + T_3 + 3}$$

(b) We are told that $t_1 = t_2 = 0.1$ so we know that

$$\frac{T_1}{T_1 + T_2 + T_3} = \frac{T_2}{T_1 + T_2 + T_3} = 0.1$$

and in particular we can conclude that $T_1 = T_2$. Also

$$T_1 = 0.1(T_1 + T_2 + T_3) = 0.1(2T_1 + T_3)$$

Multiplying by 10, we get $10T_1 = 2T_1 + T_3$ and so $T_3 = 8T_1$. The tristimulus values for color C are therefore $(T_1, T_1, 8T_1)$. So we know that T_1 is positive, because if it were negative, then all 3 tristimulus values would be negative, which can't happen (since that would mean that all 3 of the primaries get moved over to the test light side of the box, and turned up on that side). We can substitute $T_2 = T_1$ and $T_3 = 8T_1$ into the expressions for r and g :

$$r = \frac{T_1 + 1}{T_1 + T_1 + 8T_1 + 3} = \frac{T_1 + 1}{10T_1 + 3} \quad \text{and} \quad g = \frac{T_1 + 1}{T_1 + T_1 + 8T_1 + 3} = \frac{T_1 + 1}{10T_1 + 3} = r$$

So, the chromaticity coordinates for A are equal to each other, so it lies on the line $x=y$. We can say additionally that it will lie on the line segment that connects $(0.1, 0.1)$ with $(1/3, 1/3)$. To show this, we need to show

$$0.1 < \frac{T_1 + 1}{10T_1 + 3} < \frac{1}{3}$$

Since T_1 is positive, $10T_1 + 3$ is greater than zero, so we can safely multiply by $10T_1 + 3$:

$$0.1 < \frac{T_1 + 1}{10T_1 + 3} \leftrightarrow T_1 + .3 < T_1 + 1$$

which is true.

$$\frac{T_1 + 1}{10T_1 + 3} < \frac{1}{3} \leftrightarrow T_1 + 1 < \frac{10}{3}T_1 + 1$$

which is also true.

This is all you needed to show, that the point is on the line segment which connects C $(0.1, 0.1)$ and W $(1/3, 1/3)$.

One can also show that the point for A is NOT necessarily the midpoint between C $(0.1, 0.1)$ and W $(1/3, 1/3)$. To do this, take some color C which satisfies the information given. For example, tristimulus values $(0.1, 0.1, 0.8)$. This has $(r, g) = (0.1, 0.1)$. You can calculate from this that the the tristimulus values for color A are $(0.55, 0.55, 0.9)$, and the chromaticity coordinate for A are $(.275, .275)$. Next, take some other color which also satisfies the information given, for example C/2. The tristimulus values for this are $(0.05, 0.05, 0.4)$. This also has $(r, g) = (0.1, 0.1)$. But the chromaticity coordinates for A are $(.3, .3)$. In fact, if we take a sequence of colors C with tristim values $(x, x, 8x)$ as $x \rightarrow 0$, then the chromaticity coordinates for A will slide along the CW line segment going towards W.