ECE 253a	Digital Image Processing	Pamela Cosman	11/17/10

First Name: _____

Last Name: _____

ECE 253a QUIZ 2

This cover sheet is provided to give all students equal time. **Do not turn it over until everybody has received a copy of the quiz and you are instructed to start.**

You have 50 minutes to work on this quiz. You may use a calculator. You may use your class textbook, class notes, and class homeworks.

Please make sure that your copy of the quiz is complete. There are 5 problems and a total of 6 pages (including the cover sheet).

The problems are worth different numbers of points.

Problem	Possible	Score
1	6	
2	4	
3	6	
4	6	
5	4	
Total	26	

1. **Huffman Coding (6 points):**

   (a) Design a Huffman code for an alphabet with 7 symbols (A,B,C,D,E,F,G) with probabilities of occurrence 0.3, 0.26, 0.2, 0.1, 0.05, 0.05, 0.04. Show your design procedure and list the final codewords.

   (b) What is the expected length of the code?

   (c) Encode the sequence GBCEDA with your Huffman code.

   (d) Suppose your encoded sequence from part (c) is corrupted in transmission, and the first bit is flipped. What does the decoder decode?

2. **Interpolation (4 points):**

I used the notation $h_1(x)$ to denote the unit rect function which equals 1 from -0.5 to +0.5 and zero elsewhere. I defined $h_2(x) = h_1(x) * h_1(x)$; this is a triangle function, which, when used as a convolution kernel for interpolation, produces linear interpolation. Now consider

$$h_3(x) = h_1(x) * h_1(x) * h_1(x) = \begin{cases} \frac{1}{2}(x + \frac{3}{2})^2 & -\frac{3}{2} \leq x \leq -\frac{1}{2} \\ \frac{3}{4} - x^2 & -\frac{1}{2} < x \leq \frac{1}{2} \\ \frac{1}{2}(x - \frac{3}{2})^2 & \frac{1}{2} < x \leq \frac{3}{2} \end{cases}$$

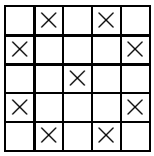
Should this h_3 kernel be called an interpolator or an approximator? That is, if a continuous function $f_I(x)$ is sampled at integer locations, and the sampled function is then convolved with h_3 to produce the reconstructed function $f_R(x)$, will the reconstructed function be exactly equal to the original function at the sample points?

3. **Median Filtering (6 points):**

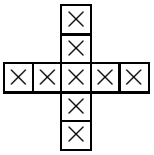
The three median filters shown below have the same complexity (as they all involve putting 9 numbers in sorted order to compute the median) but they differ in other ways. Briefly discuss the advantages and disadvantages of these 3 median filters in terms of (a) ability to preserve thin lines and corners, (b) ability to remove noise which occurs as 3x3 noise blocks, and (c) ability to filter a noise-free image and leave it as unchanged as possible.

9-point square MF:

9-point sparse MF:



9-point plus-shaped MF:



4. **Scalar Quantization – optimality conditions (6 points)**

In this problem, the distortion measure is mean-squared error. Suppose that a random variable X has the two-sided exponential pdf

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$$

A three level quantizer q for X has the form

$$q(x) = \begin{cases} +b & x > a \\ 0 & -a \leq x \leq +a \\ -b & x < -a \end{cases}$$

(a) Find an expression for b as a function of a so that the centroid condition is met. Your expression may involve integrals and you do NOT have to carry out the integration.

(b) Find a simple expression (no integrals) for a as a function of b so that the nearest neighbor condition for optimality is satisfied.

5. **Vector Quantization (4 points):**

   (a) Sketch the region in the 2D plane consisting of all points having Euclidean distance less than 2 from the origin.

   (b) Sketch the region in the 2D plane consisting of all points having a city-block distance less than 2 from the origin.