

COLOR

The figures in this handout are taken from *Foundations of Vision* by Brian Wandell, *Color in Electronic Displays* by Heino Widdel and David Post, *Digital Image Processing* by William Pratt, and *Fundamentals of Digital Image Processing* by Anil Jain.

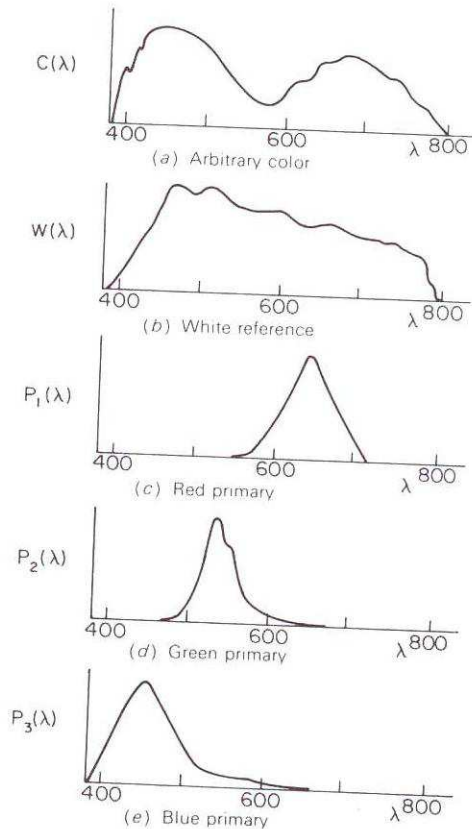
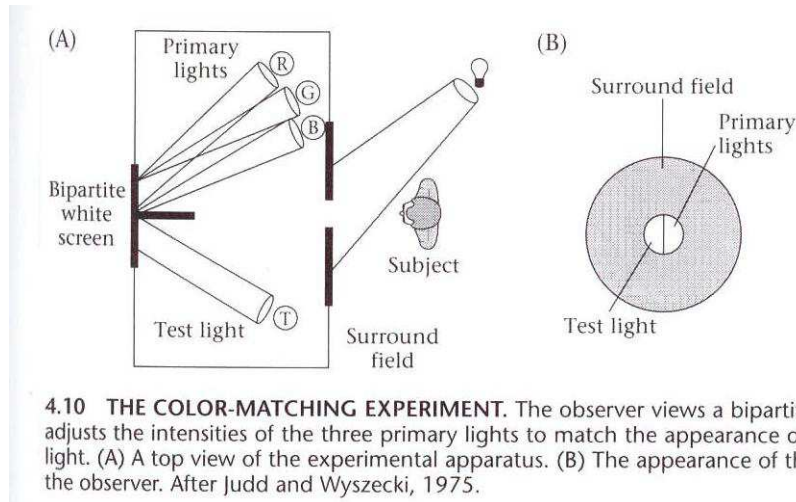
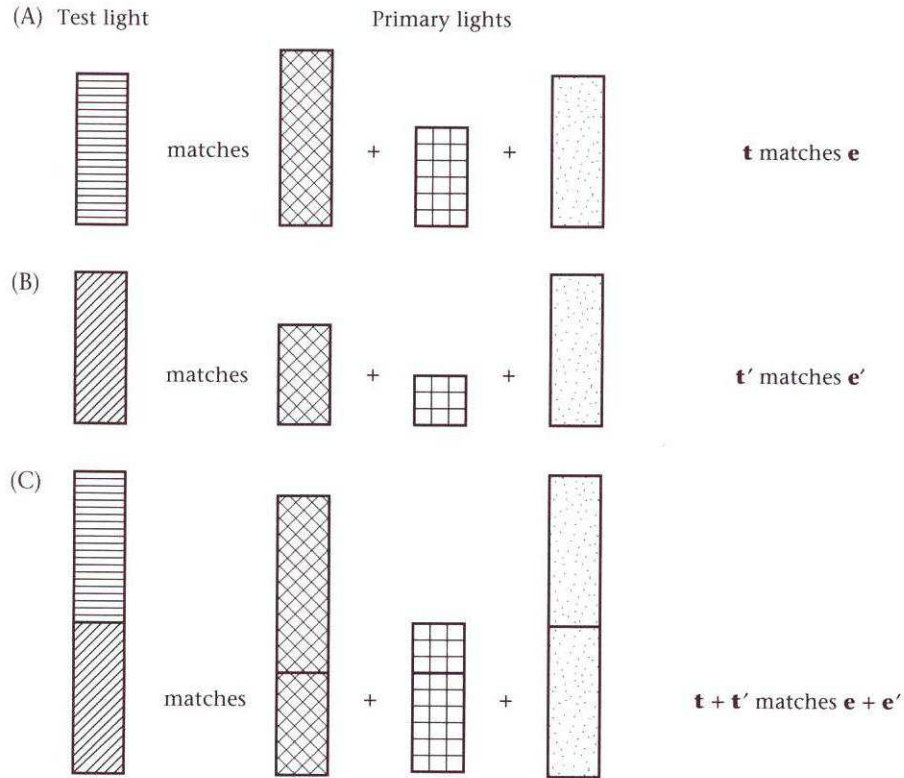


FIGURE 3.2-2. Spectral energy distributions.



4.12 THE COLOR-MATCHING EXPERIMENT SATISFIES THE PRINCIPLE OF SUPERPOSITION. In parts (A) and (B), test lights are matched by a mixture of three primary lights. In part (C) the sum of the test lights is matched by the additive mixture of the primaries, demonstrating superposition.

Algebraic formulation of color

Suppose we have some arbitrary light $C(\lambda)$. We consider that λ is sampled e.g., 380–780 nm sampled at 2 nm intervals, so we get 201 numbers, or 400–700 nm sampled at 10 nm intervals, so 31 numbers. We can represent C as

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

So we can write this as:

$$c = c_1 e_1 + c_2 e_2 + \dots + c_n e_n$$

where the e_i are the spectral (prism) colors:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \dots \quad e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Now we have some primaries. Call them P_1, P_2, P_3 . Each of these is itself a column vector of length n . With these primaries, we use the color matching setup to match the spectral colors.

$$m_{1i}P_1 + m_{2i}P_2 + m_{3i}P_3 \leftrightarrow e_i$$

The symbol \leftrightarrow means “visual match.” It is supposed to be used for two column vectors, and means that the two spectral distributions over wavelengths of visible light look the same to the human visual system. This equation can be written also as

$$\left[\begin{array}{c|c|c} P_1 & P_2 & P_3 \\ \hline \end{array} \right] \left[\begin{array}{c} m_{1i} \\ m_{2i} \\ m_{3i} \end{array} \right] \leftrightarrow \left[\begin{array}{c} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{array} \right]$$

or we can write it in shorthand as $PM_i \leftrightarrow e_i$

where P is the $n \times 3$ matrix made up of the 3 column vectors of the primaries:

$$P = \left[\begin{array}{c|c|c} P_1 & P_2 & P_3 \\ \hline \end{array} \right]$$

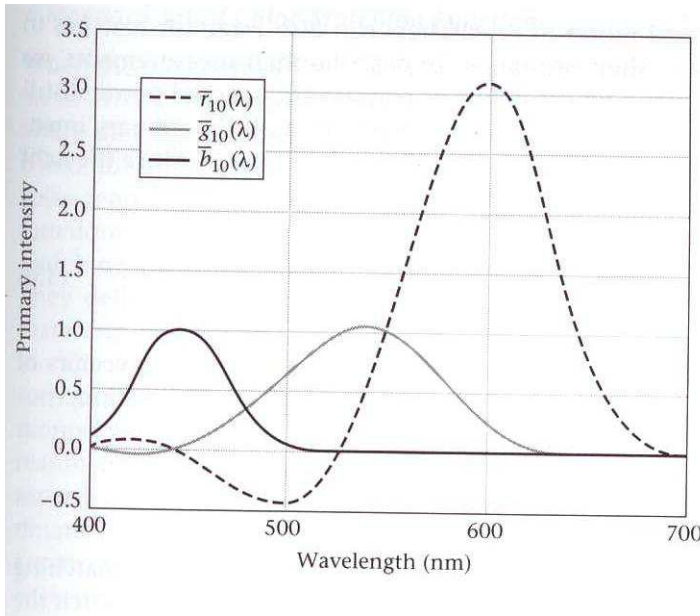
After this matching is done for all spectral colors, you get

$$PM \leftrightarrow I$$

This last equation is something of an abuse of the \leftrightarrow notation. When the symbol is used for matrices, we mean that the j th column on the left is a visual match for the j th column on the right. $PM \leftrightarrow I$ is shorthand for:

$$\left[\begin{array}{c|c|c} P_1 & P_2 & P_3 \\ \hline \end{array} \right] \left[\begin{array}{ccc} m_{11} & m_{1i} & \\ m_{21} & \dots & m_{2i} & \dots \\ m_{31} & & m_{3i} & \end{array} \right] \leftrightarrow \left[\begin{array}{cccc} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{array} \right]$$

The COLOR MATCHING FUNCTIONS are the rows of the color matching system matrix M :



4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX. The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called $\bar{r}_{10}(\lambda)$, $\bar{g}_{10}(\lambda)$, and $\bar{b}_{10}(\lambda)$.

Then for any color

$$c = c_1 e_1 + c_2 e_2 + \dots + c_n e_n$$

using the axioms of “homogeneity” and additivity, we have

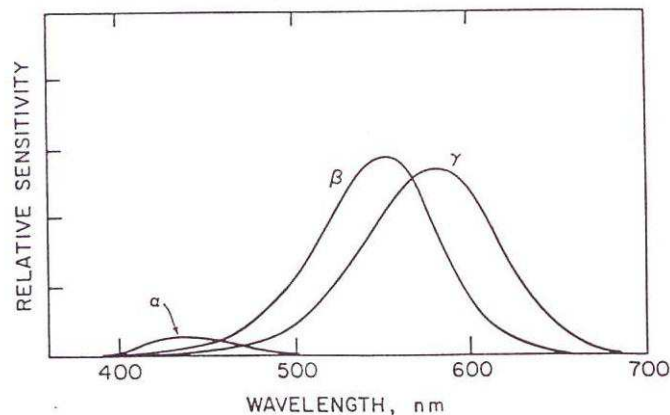
$$PMc \leftrightarrow c$$

If we denote $Mc = m_c$ then we have

$$Pm_c \leftrightarrow c$$

This means that with three primaries, we can match any color.

Why should 3 primaries let you match any color? Because there are 3 basic types of cones in the retina. They have different absorption characteristics as a function of wavelength:



Typical spectral absorption curves of pigments of the retina

So one can write the neural response as $r = Rc$ where this is shorthand for

$$\begin{array}{ccccc} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} & = & \begin{bmatrix} - & R_1 & - \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix} & \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \\ \uparrow & & \uparrow & & \uparrow \\ \text{neural} & & \text{spectral} & & \text{color signal} \\ \text{response} & & \text{sensitivity} & & \\ & & \text{of } i\text{th cone type} & & \end{array}$$

Two colors c_A and c_B match if

$$r = Rc_A = Rc_B \Leftrightarrow c_A \leftrightarrow c_B$$

$$R(c_A - c_B) = 0$$

or $c_A - c_B$ is in the nullspace of R then colors match.

$$c \leftrightarrow \begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} m_{c1} \\ m_{c2} \\ m_{c3} \end{bmatrix} = Pm_c$$

Since c matches Pm_c , we have

$$R(c - Pm_c) = 0 \quad \Rightarrow Rc = RPm_c \quad \Rightarrow m_c = (RP)^{-1}Rc$$

m_c is 3×1

$(RP)^{-1}$ is 3×3

R is $3 \times n$

c is $n \times 1$

Apply this to spectral color e_i

$$\text{Since } PM_i \leftrightarrow e_i \text{ therefore } M_i = (RP)^{-1}Re_i$$

Do this for all the e_i and we get

$$M = (RP)^{-1}RI = (RP)^{-1}R$$

$(RP)^{-1}$ is a 3×3 linear transformation.

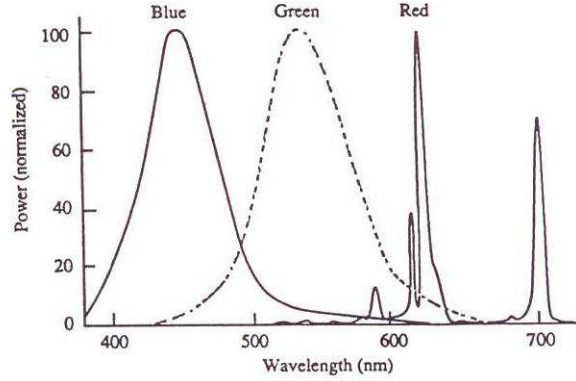
So this means that the color matching functions are within a linear transformation of our receptor responses.

Matching a color on our monitor:

We see some color c and wish to reproduce this on our monitor. The neural response that we wish to replicate is $r = Rc$. Let

$$D = \begin{bmatrix} | & | & | \\ D_1 & D_2 & D_3 \\ | & | & | \end{bmatrix}$$

be the spectral power distribution of the phosphors of our display.



$$g = [g_1 g_2 g_3]^t$$

are the gun intensities. Then the neural response is

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} - & R_1 & - \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix} \begin{bmatrix} | & | & | \\ D_1 & D_2 & D_3 \\ | & | & | \end{bmatrix} g$$

We choose g to match that light:

$$RDg = Rc$$

$$g = (RD)^{-1} Rc$$

$$0 \leq g \leq Max$$

constrains the colors that you can actually match.

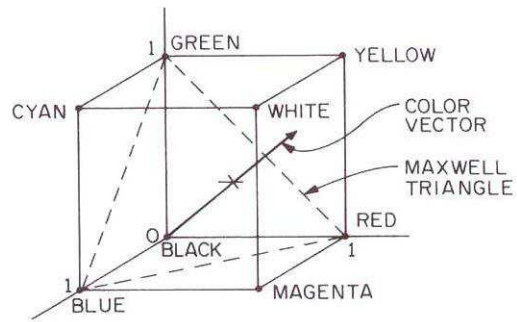


FIGURE 3.3-2. Color space for typical red, green and blue primaries.

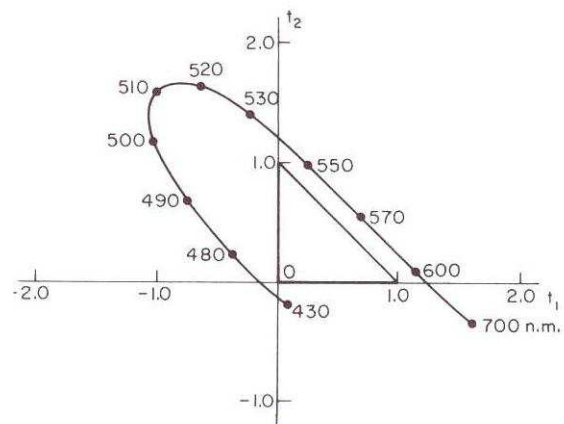


FIGURE 3.3-3. Chromaticity diagram for typical red, green and blue primaries.

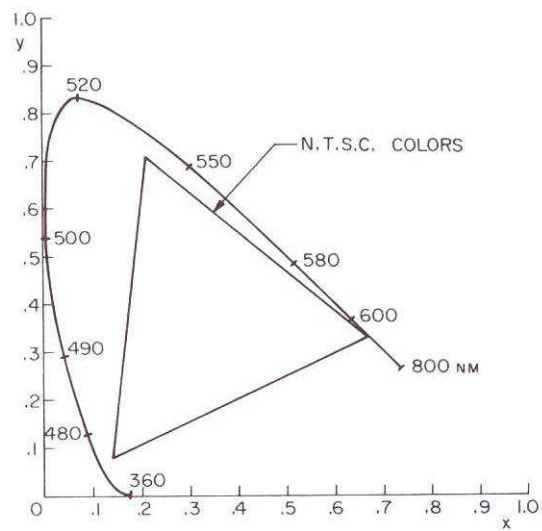
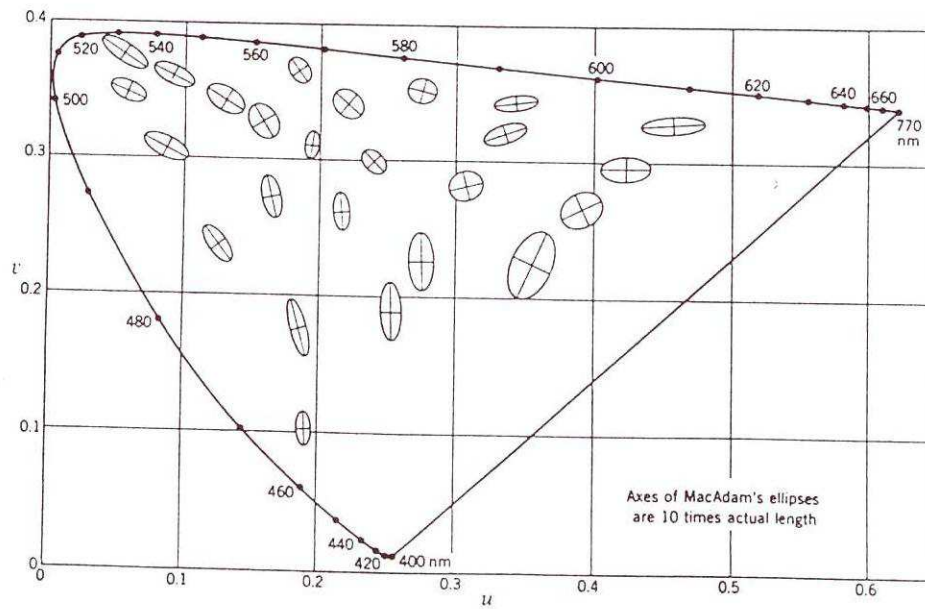
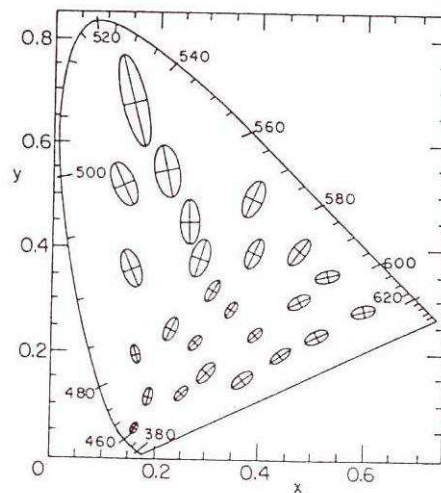


FIGURE 3.5-5. Chromaticity diagram for CIE XYZ primary system.



(b) $u-v$ chromaticity diagram

FIGURE 3.7-2. MacAdam's ellipses of just noticeable color differences in XYZ and UVW coordinate systems (39). Axes of ellipses are 10 times actual length.



(a) $x-y$ chromaticity diagram

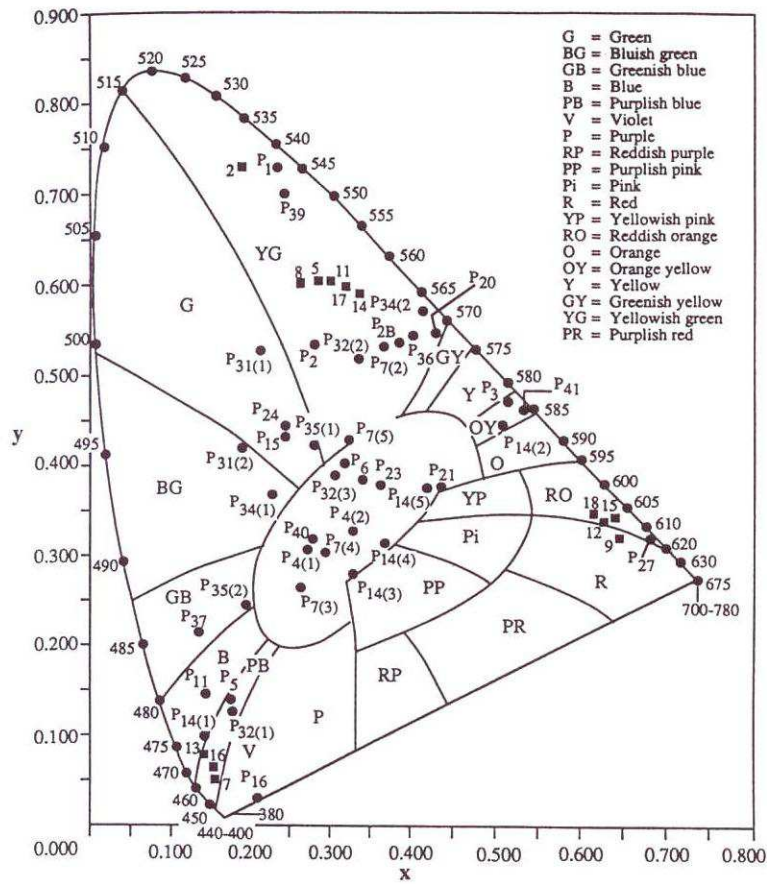


Figure 10. Plot of common CRT phosphors in the CIE 1931 (x,y)-chromaticity diagram. Circles represent phosphors intended for monochrome displays whereas squares represent varieties of the P22 phosphor family, which are intended for color displays. From Laycock, J., & Viveash, J.P. (1982). Calculating the perceptibility of monochrome and color displays viewed under various illumination conditions.