HOMEWORK 2

Due Wed January 30 in class

1. **Median Filtering**

   (a) On the web site, you will find an original image named pep.tif and a noisy version of it called n2.tif. The noisy version was made from pep using the following commands:

   ```
   noi = rand(256);
   n2 = zeros(size(pep));
   n2 = n2 + pep .* (noi >0.2) + 255 .* (noi<0.1);
   ```

   What kind of noise is this?

   (b) Write a routine that performs two-dimensional median filtering with the 5-point cross-shaped median filter. Use your median filter to clean it up.

   Also you can use the command medfilt2 to do median filtering with different size filters. In addition to your cross-shape, try the following sizes: 1x2 1x3 2x2 3x3 3x4 4x4 4x5 5x5 7x7

   If a median filter were able to perfectly clean up the noisy image, then the mean-squared error between the cleaned-up version and the original image would be zero. In practice, this doesn't happen. We can use the mean-squared error as a simple measure of how well a given median filter is doing. Plot the mean-squared error of the result versus the number of points in the filter for all the filters above, and for the cross-shaped filter. Include the mean-squared error between pep and n2 in your plot as well, as being unfiltered (a one-point median filter).

   Looking at this plot for the n2.tif image, explain what you see. That is, explain the ups and downs in the plot, and what is the best filter.

   (c) Another noisy version of the image is n4.tif. Here the noise is twice as severe. Make the same kind of plot for this image. Compare the plots for n2 and n4: what can you say about them? In particular, explain the overall difference in MSE and the difference in the location of the minimum.

   (d) A different original image is bab.tif. It's noisy version is bab2.tif, which has the same amount of noise as n2 does. Make the same kind of plot for this image. Compare the plots for n2 and bab2: what can you say about them?

2. **Filtering to Fix a Corrupted Image**

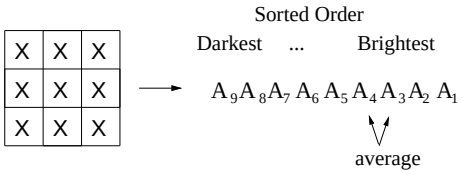
The original airplane image (shown on the left) has been corrupted. Every fifth column has been set to zero, as shown on the right.

We propose 7 filtering approaches to fix this. For each, the proposed filter will only be applied to pixels in the corrupted columns (every 5th column). Pixels not in the corrupted columns are identical to the original image and will be left unchanged. Rank the 7 approaches in terms of what you think the mean squared error (MSE) would be between the "fixed" image and the original. If any proposed method would give exactly the same MSE as some other method, say that. Ignore boundary effects such as zero-padding at the edges. Give an explanation for your ranking.

(a) Horizontal 3-point median filter
(b) Horizontal 3-point mean filter
(c) Horizontal 3-point midpoint filter
(d) 6-point sparse median filter, as shown. The output, corresponding to the center point of the filter, is the median of the 6 points in the filter marked with an X. Note that the pixel itself is not used in the median computation.

X		X
X		X
X		X

(e) 5-point plus-shaped median filter
(f) 3x3 square 67% order-statistic filter. This filter puts the 9 elements in a 3x3 square in sorted order, and then takes the average of the 3rd and 4th brightest ones.

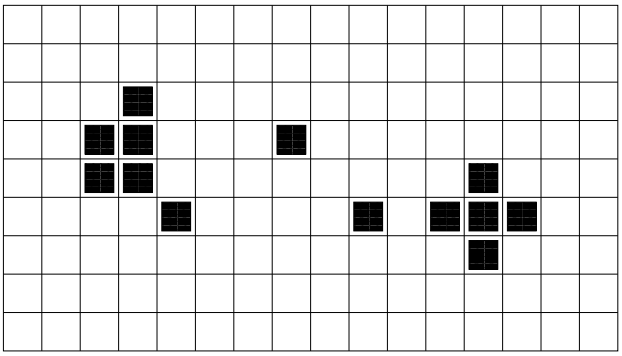


(g) 3x3 weighted median filter which uses the following weighting

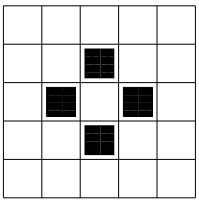
1	3	1
1	3	1
1	3	1

3. **Binary Morphological Operators**

(a) The image X below (of size 16 pixels horizontally and 9 pixels vertically) contains four separate 8-connected blobs. Find the smallest structuring element B so that $X \oplus B$ (one dilation operation) consists of just a single 8-connected blob. B must be either 4- or 8-connected. Smallest means least number of pixels in B.

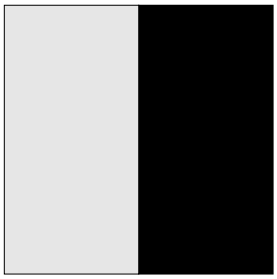


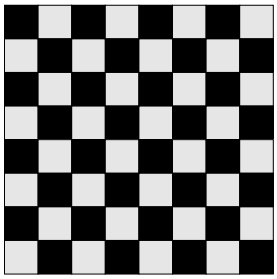
(b) This part is unrelated to the previous part. Consider the set A below with 4 pixels. For some structuring elements B which contain exactly two pixels, $A \bullet B = A$. Give an example of a B with only 2 pixels (label it B_T) for which this equation is true. Give an example of a B with only 2 pixels (label it B_F) for which this equation is false.



4. **Histograms and Averaging:**

The images shown below, of size 200 by 200 pixels, are quite different, but their histograms are the same. Suppose that each image is blurred with a 3×3 spatial averaging mask.

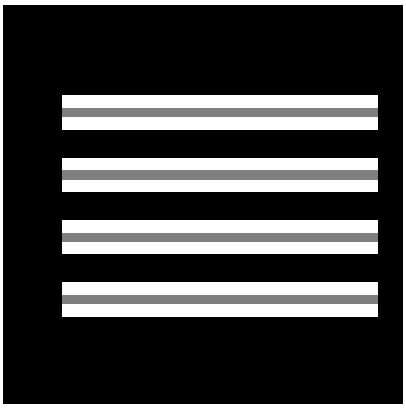




(a) Would the histograms of the blurred images still be equal?
(b) If your answer is no, sketch the two histograms.

5. **Resolution of a Median Filter**

Here is an original image. It has only 3 gray levels. The middle gray portion of each bar has height 3 pixels. The white bars on each side have height 4 pixels.



Median filtering was applied to this image, with median filters of sizes 5x5, 15x15, and 31x31. The results are shown below. Which one is which?