# 19	ECE 253a	Digital Image Processing	Pamela Cosman	11/19/10

QUIZ 2 SOLUTIONS

1. **Huffman Coding:**

(a) One possible codeword assignment is:

Symbol	Codeword Length	Codeword
A	2	11
B	2	10
C	2	01
D	3	001
E	4	0001
F	5	00001
G	5	00000

(Other correct answers are also acceptable.)

(b) The expected length of this code is:

$$E(l) = 0.3 \times 2 + 0.26 \times 2 + 0.2 \times 2 + 0.1 \times 3 + 0.05 \times 4 + 0.05 \times 5 + 0.04 \times 5 = 2.47 bits$$

c)GBCEDA is encoded to the sequence of 000001001000100111

d)If we flip the first bit of the sequence, we get 10,0001,001,0001,001,11 and is decoded to BEDEDA, and we see three symbol errors.

Grading Criteria: 3 points for part (a), 1 point for each of the other parts.

2. **Interpolation:**

$h_3(0) = \frac{3}{4}$, and $h_3(-1) = h_3(1) = \frac{1}{8}$. The reconstructed function sampled at 0 is:

$$f_R(0) = \frac{3}{4} \times f_I(0) + \frac{1}{8} \times f_I(-1) + \frac{1}{8} \times f_I(1)$$

which is not always equal to $f_I(0)$, so $h_3(x)$ is an approximator.

3. **Median Filtering:**

(a) A line of width 1 running in any direction will be completely eliminated by the 3x3 square filter and by the sparse filter. It will be left untouched by the other filter. Consider now a corner, that is, where you have one quadrant with value a, and the other 3 quadrants of the plane with value b. This is a root signal for the plus-shaped filter, so it retains it perfectly. The 3x3 square will eat away the corner of the a quadrant. The sparse filter does the worse. It will eat away three pixels from the corner.

(b) The 3x3 square filter will eat away the four corners of the 3x3 square noise block. The other 5 pixels of the noise block will remain. The sparse filter will remove the noise block entirely. The plus-shaped filter will have no effect on the 3x3 noise block (it is a root signal).

(c) Other than the issue of preserving thin lines and corners, the 3x3 square filter is the best choice if you're forced to do noise filtering on a noise-free image, because it is taking values from nearby, so will in general be changing the noise-free pixels the least. The plus-shaped filter would be next best, and the sparse filter would be the worst, because it is taking pixels from farther away.

Grading Criteria: 2 points for each part. Partial credit if answer is partly right.

4. **Scalar Quantization – optimality conditions**

   (a) The centroid condition stipulates that b should be the centroid of the probability that is mapping into that region. Looking only at positive x:

$$b = \frac{\int_a^\infty x\frac{\lambda}{2}e^{-\lambda x}dx}{\int_a^\infty \frac{\lambda}{2}e^{-\lambda x}dx}$$

   This is all you need to say for full credit for this part. Although you don't need to do this, to evaluate the integral: after eliminating the $\lambda/2$ on top and bottom, we integrate the numerator by parts to obtain

$$numerator = \frac{ae^{-\lambda a}}{\lambda} + \frac{e^{-\lambda a}}{\lambda^2}$$

   and

$$denominator = \frac{e^{-\lambda a}}{\lambda}$$

$$b = a + \frac{1}{\lambda}$$

   (b) The other condition for optimality says that the decision level must be halfway between the reconstruction levels. So

$$a = \frac{b}{2}$$

   This is all you need to say for full credit for this part. Therefore, the 3-level quantizer is characterized by

$$a = \frac{1}{\lambda} \qquad \text{and} \qquad b = \frac{2}{\lambda}$$

Grading Criteria: 3 points for each part. Partial credit allowed.

5. **Vector Quantization**

(a)The region having Euclidean distance less than 2 is defined by a disk of:

$$D = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 < 4\}$$

(b)The region with city-block distance less than 2 is defined by a diamond of:

$$D = \{(x, y) \in \mathbb{R}^2 | \, |x| + |y| < 2\}$$

Grading criteria: 2 points for each part.