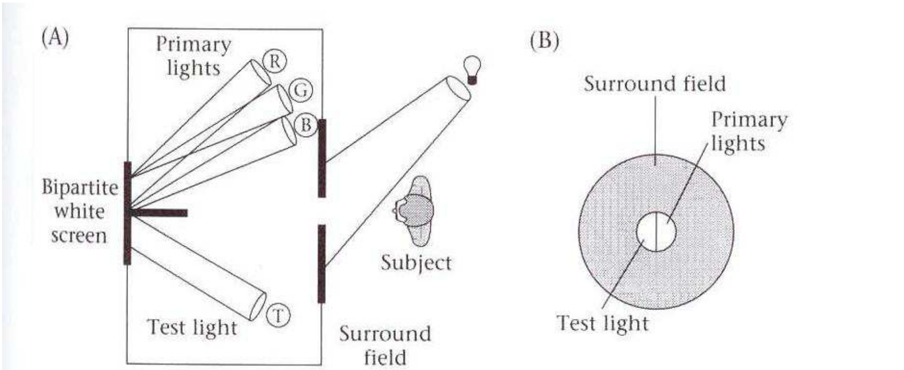
# 11	ECE 172a	Pamela Cosman	2/6/12

COLOR

The figures in this handout are taken from *Foundations of Vision* by Brian Wandell, *Color in Electronic Displays* by Heino Widdel and David Post, *Digital Image Processing* by William Pratt, and *Fundamentals of Digital Image Processing* by Anil Jain.



4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

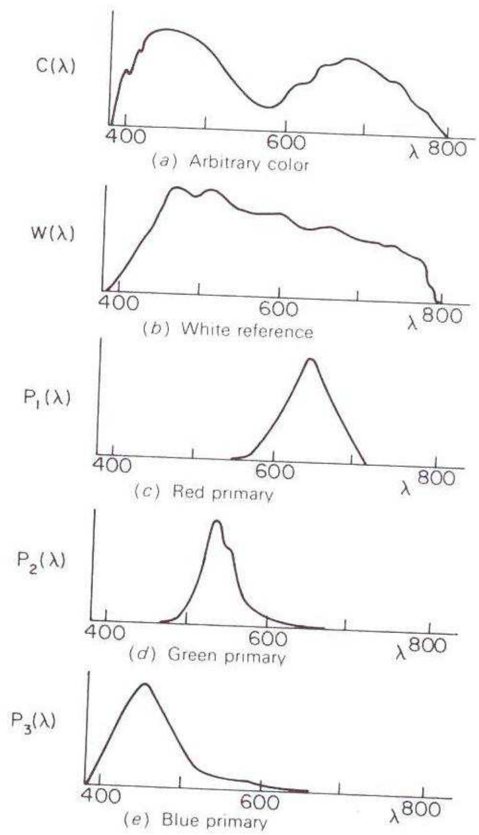
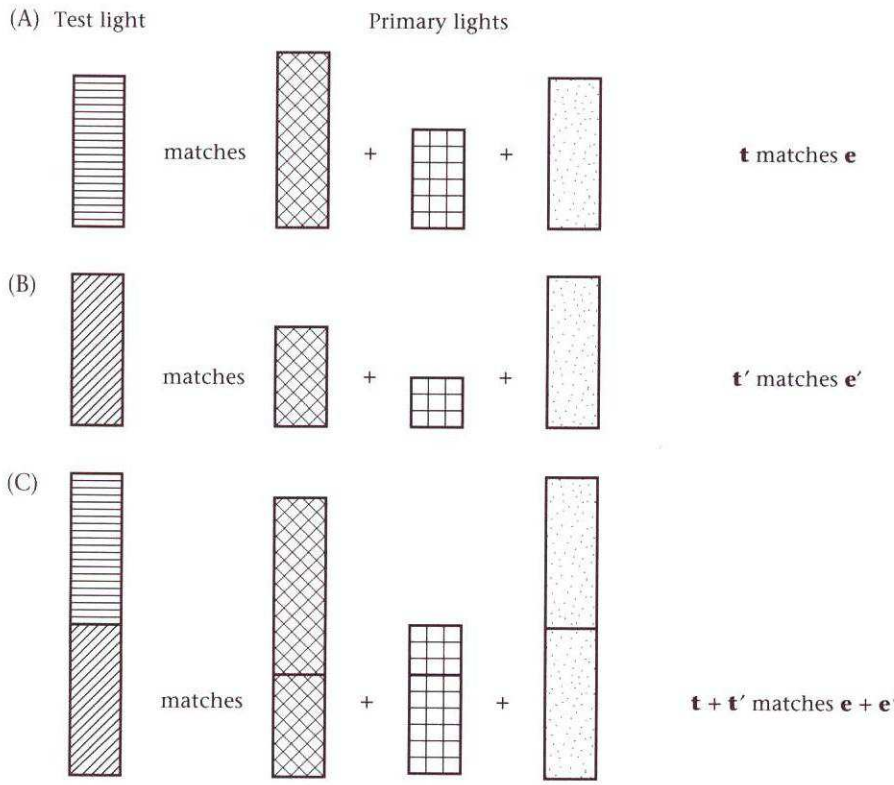


FIGURE 3.2-2. Spectral energy distributions.



4.12 THE COLOR-MATCHING EXPERIMENT SATISFIES THE PRINCIPLE OF SUPERPOSITION. In parts (A) and (B), test lights are matched by a mixture of three primary lights. In part (C) the sum of the test lights is matched by the additive mixture of the primaries, demonstrating superposition.

Algebraic formulation of color

Suppose we have some arbitrary light $C(\lambda)$. We consider that λ is sampled e.g., 380–780 nm sampled at 2 nm intervals, so we get 201 numbers, or 400–700 nm sampled at 10 nm intervals, so 31 numbers. We can represent C as

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

So we can write this as:

$$c = c_1 e_1 + c_2 e_2 + \ldots + c_n e_n$$

where the e_i are the spectral (prism) colors:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \ldots \quad e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Now we have some primaries. Call them P_1, P_2, P_3. Each of these is itself a column vector of length n. With these primaries, we use the color matching setup to match the spectral colors.

$$m_{1i}P_1 + m_{2i}P_2 + m_{3i}P_3 \leftrightarrow e_i$$

The symbol $\leftrightarrow$ means "visual match." It is supposed to be used for two column vectors, and means that the two spectral distributions over wavelengths of visible light look the same to the human visual system. This equation can be written also as

$$\begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} m_{1i} \\ m_{2i} \\ m_{3i} \end{bmatrix} \leftrightarrow \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or we can write it in shorthand as $PM_i \leftrightarrow e_i$
where P is the $n \times 3$ matrix made up of the 3 column vectors of the primaries:

$$P = \begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix}$$

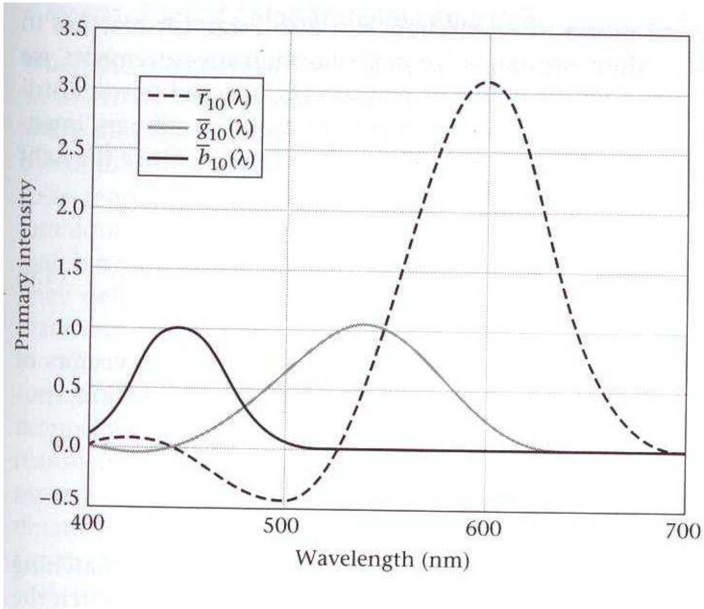
After this matching is done for all spectral colors, you get

$$PM \leftrightarrow I$$

This last equation is something of an abuse of the $\leftrightarrow$ notation. When the symbol is used for matrices, we mean that the jth column on the left is a visual match for the jth column on the right. $PM \leftrightarrow I$ is shorthand for:

$$\begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} m_{11} & & m_{1i} & \\ m_{21} & \dots & m_{2i} & \dots \\ m_{31} & & m_{3i} & \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

The COLOR MATCHING FUNCTIONS are the rows of the color matching system matrix M:



4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX. The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called $\bar{r}_{10}(\lambda)$, $\bar{g}_{10}(\lambda)$, and $\bar{b}_{10}(\lambda)$.

Then for any color

$$c = c_1 e_1 + c_2 e_2 + \ldots + c_n e_n$$

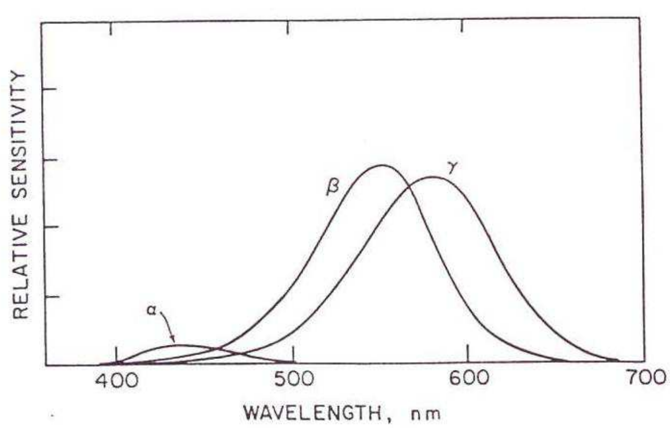
using the axioms of “homogeneity” and additivity, we have

$$PMc \leftrightarrow c$$

If we denote $Mc = m_c$ then we have

$$Pm_c \leftrightarrow c$$

This means that with three primaries, we can match any color.
Why should 3 primaries let you match any color? Because there are 3 basic types of cones in the retina. They have different absorption characteristics as a function of wavelength:



Typical spectral absorption curves of pigments of the retina

So one can write the neural response as $r = Rc$ where this is shorthand for

$$\underset{\substack{\big\uparrow \\ \text{neural} \\ \text{response}}}{\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}} = \underset{\substack{\big\uparrow \\ \text{spectral} \\ \text{sensitivity} \\ \text{of } i\text{th cone type}}}{\begin{bmatrix} - & R_1 & - \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix}} \underset{\substack{\big\uparrow \\ \text{color signal}}}{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}}$$

Two colors c_A and c_B match if

$$r = Rc_A = Rc_B \Leftrightarrow c_A \leftrightarrow c_B$$

$$R(c_A - c_B) = 0$$

or $c_A - c_B$ is in the nullspace of R then colors match.

$$c \leftrightarrow \begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} m_{c1} \\ m_{c2} \\ m_{c3} \end{bmatrix} = Pm_c$$

Since c matches Pm_c, we have

$$R(c - Pm_c) = 0 \qquad \Rightarrow Rc = RPm_c \qquad \Rightarrow m_c = (RP)^{-1}Rc$$

m_c is 3×1
$(RP)^{-1}$ is 3×3
R is $3 \times n$
c is $n \times 1$

Apply this to spectral color e_i

$$\text{Since } PM_i \leftrightarrow e_i \text{ therefore } M_i = (RP)^{-1}Re_i$$

Do this for all the e_i and we get

$$M = (RP)^{-1}RI = (RP)^{-1}R$$

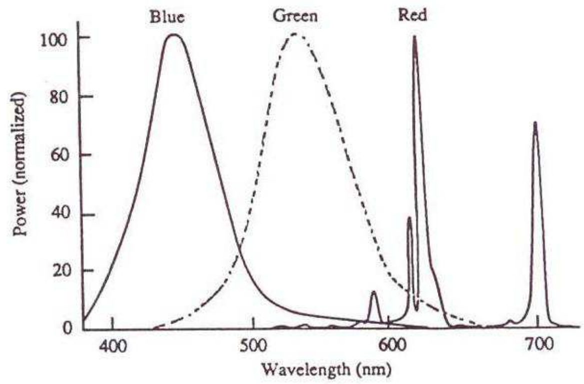
$(RP)^{-1}$ is a 3×3 linear transformation.
So this means that the color matching functions are within a linear transformation of our receptor responses.

Matching a color on our monitor:

We see some color c and wish to reproduce this on our monitor. The neural repsonse that we wish to replicate is $r = Rc$. Let

$$D = \begin{bmatrix} | & | & | \\ D_1 & D_2 & D_3 \\ | & | & | \end{bmatrix}$$

be the spectral power distribution of the phosphors of our display.



$$g = [g_1 g_2 g_3]^t$$

are the gun intensities. Then the neural response is

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} - & R_1 & - \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix} \begin{bmatrix} | & | & | \\ D_1 & D_2 & D_3 \\ | & | & | \end{bmatrix} g$$

We choose g to match that light:

$$RDg = Rc$$

$$g = (RD)^{-1} Rc$$

$$0 \leq g \leq Max$$

constrains the colors that you can actually match.

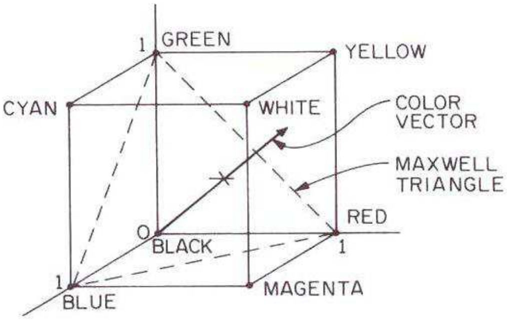


FIGURE 3.3-2. Color space for typical red, green and blue primaries.

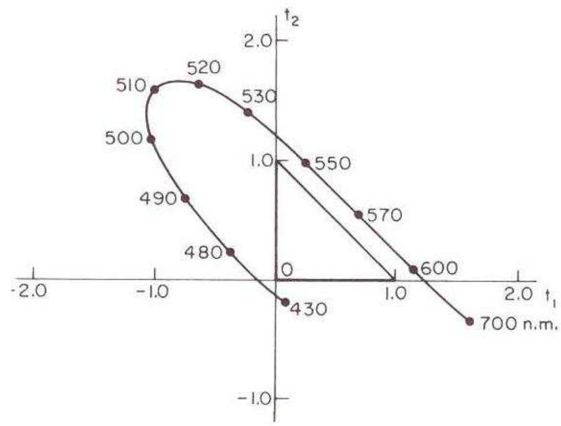


FIGURE 3.3-3. Chromaticity diagram for typical red, green and blue primaries.

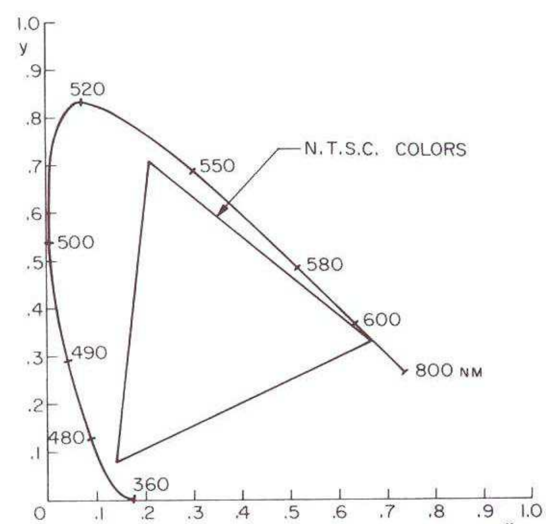
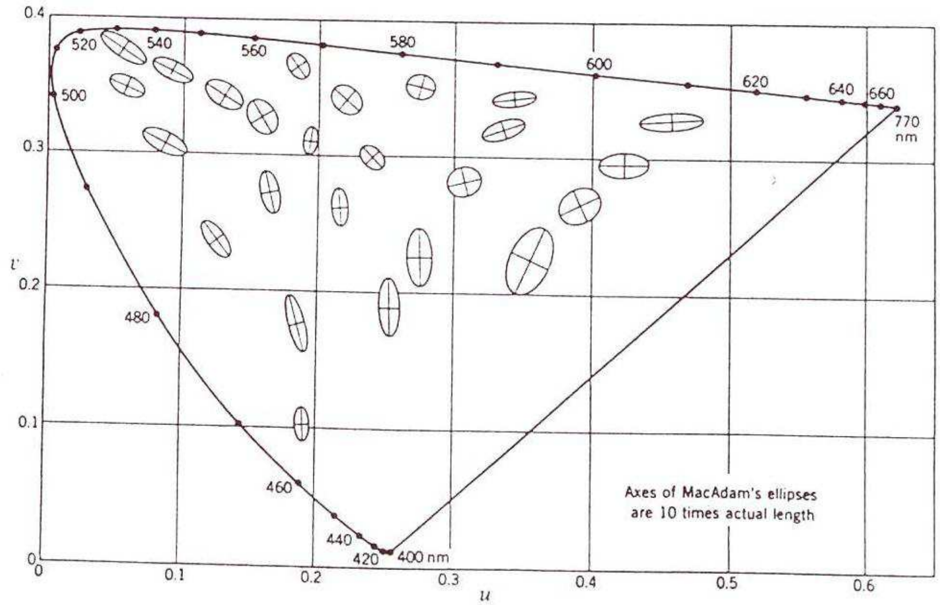
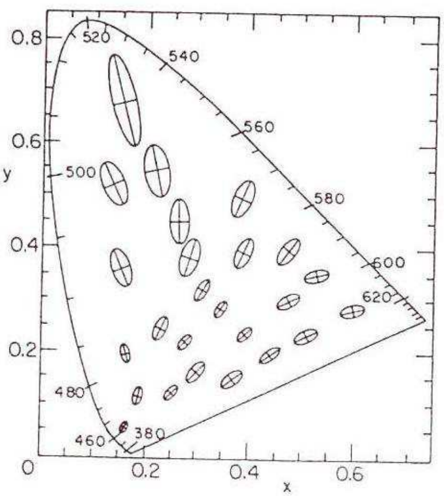


FIGURE 3.5-5. Chromaticity diagram for CIE *XYZ* primary system.



(*b*) *u–v* chromaticity diagram

FIGURE 3.7-2. MacAdam's ellipses of just noticeable color differences in *XYZ* and *UVW* coordinate systems (39). Axes of ellipses are 10 times actual length.



(*a*) *x–y* chromaticity diagram