Wireless Broadcasting Using Network Coding

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Abstract— **Traditional approaches to transmit information reliably over an error-prone network employ either Forward Error Correction (FEC) or retransmission techniques. In this paper we consider an application of network coding to increase the bandwidth efficiency of reliable broadcast in a wireless network. In particular, we propose two schemes which employ network coding to reduce the number of retransmissions as a result of packet losses. Our proposed schemes combine different lost packets from different receivers in such a way that multiple receivers are able to recover their lost packets with one transmission by the source. The advantages of the proposed schemes over the traditional wireless broadcast are shown through simulations and theoretical analysis. Specifically, we provide a few results on the retransmission overhead of the proposed schemes under different channel conditions.**

I. Introduction

Broadcast is a mechanism for disseminating identical information from one source to many receivers. It is widely used in many applications ranging from satellite communications to wireless mobile ad hoc networks. Reliable broadcast requires that every receiver must receive the correct information sent by the source. When the communication channels between a source and receivers are lossy, the appropriate schemes must be used to provide reliable transmissions. Depending on the applications, these schemes can be classified into two main approaches: retransmission and Forward Error Correction (FEC). Using retransmission approach, the source simply rebroadcasts the lost data if there is at least one receiver not receiving the correct data. This approach assumes that the receivers can somehow communicate to the source whether or not it receives the correct data. On the other hand, using the FEC approach, the source encodes additional information together with the data before broadcasting them to the receivers. If the amount of lost data is sufficiently small, a receiver can recover the lost data using some decoding schemes. For satellite TV applications, the TV signals are broadcast from a satellite to potentially hundreds of millions TVs, and thus the probability of any TVs not receiving the correct signal at any time is close to 1. In this scenario, retransmission of the lost data to every TV is therefore prohibitive due to the large satellite bandwidth requirement and the lack of a reverse TV-to-satellite channel.

On the other hand, in a wireless home network, there are relatively few devices and the communication channel is relatively reliable. Therefore, the retransmission approach may be more bandwidth efficient than that of FEC since redundant information is not added in every transmission. Similarly, in a wireless ad hoc network, routing information is often broadcast to a relatively few number of neighbor nodes, and thus retransmission may be used. That said, the focus of this paper will be on combining network coding and retransmission to efficiently utilize the broadcast bandwidth.

Network coding is a new approach to increase the transmission capacity of a network [1]. In a traditional store-and-forward network, packets are forwarded hop-by-hop along the intermediate nodes (e.g. routers) from a source to a destination. An intermediate node forwards the packets as it receives through a predefined path. On the other hand, network coding techniques allow an intermediate node to combine data from different input links before sending the combined data on its output links. This is called network encoding. For many problems such as multicast and broadcast, using appropriate encoding schemes at each intermediate nodes (typically linear combination of input data) can achieve the network capacity. Network coding technique can also be applied to wireless networks [2][3]. Section II provides a few representative works in this area.

Similar to [2][4] (as discussed in Section II), our proposed schemes combine different lost packets from different receivers in such a way to allow multiple receivers recover their lost packets simultaneously with one transmission from the source. Specifically, in the proposed schemes, the source does not retransmit the lost packet immediately when it receives a negative acknowledgment (NAK). Instead, the retransmission phase starts at a fixed interval of time. This technique enables the source to combine lost packets from different receivers into one packet. We show that this approach can reduce the transmission bandwidth significantly.

The organization of our paper is as follows. We first discuss a few related work in Section II. In Section III, we describe different *retransmission* broadcast schemes with and without network coding. We then analyze the performance in terms of bandwidth utilization for these schemes in Section IV. In particular, we derive a few results showing the reduced transmission bandwidth when network coding is employed. Section V shows the simulation results that confirm our theoretical predictions.

II. RELATED WORK

Information broadcast in a network is a well explored problem. Many proposed schemes aim to minimize the energy or bandwidth efficiency in different types of networks have been proposed. Cagalj et al. prove that minimum-energy broadcast (without network coding) for arbitrary network topologies is an *NP-complete* problem [5]. Recently, network coding approaches to wireless network have shown promising schemes for reducing the energy and bandwidth. Fragouli et al. provides an overview of network coding and its application in wireless networks [6]. Wu et al. apply network coding for information exchange of independent information in wireless networks [2]. Similar framework is proposed in [4]. These schemes show that the information exchange between two wireless nodes through an intermediate node can be performed efficiently using XOR operations, a form of network coding. In the XOR schemes, two nodes R_1 and R_2 are assumed to exchange information through a node R_3. A packet a sent by node R_1 to node R_2 is relayed by node R_3. Similarly, packet b sent by node R_2 to node R_1 is relayed by node R_3. As a result, node R_3 has both packets a and b. Traditionally, node R_3 has to perform two transmissions for packets a and b. On the other hand, since node R_1 already has packet a, and node R_2 already has packet b, node R_3 can simply broadcast a single packet $a \oplus b$ to both nodes R_1 and R_2. Upon receiving this packet, node R_1 can obtain the packet b as $b = a \oplus (a \oplus b)$. Node R_2 can also recover packet a as $b = b \oplus (a \oplus b)$. Our proposed broadcast scheme employs a similar technique as will be discussed in Section III. In their other work, Wu et al. propose network coding approach for multicast information in wireless ad-hoc networks to achieve minimum energy consumption [3]. Recently, J. Widmer et al. employ network coding technique to broadcast information in wireless ad hoc networks where a large percentages of the nodes act as sources [7]. They provide a low-complexity distributed algorithms which can reduce the average energy consumption of a network.

The work that is most related ours is that of Eryilmaz et al. [8]. In [8], the authors propose some random network coding schemes for wireless broadcast of multiple files. Rather than using XOR operations, their schemes code every packet and use a large size field to guarantee decodability. The authors also provide different algorithms and show their performances when the channel state is available and when it is not.

III. BROADCAST SCHEMES

Before describing different schemes, we make the following assumptions for all the broadcast schemes:

1) There is one source and $M > 1$ receivers.
2) Data is assumed to be sent in packets, and each packet is sent in a time slot of fixed duration.
3) The source assumes to know which packet from which receiver is lost. This can be accomplished through the use of positive and negative acknowledgments (ACK/NAKs). For simplicity, we assume all the ACK/NAKs are instantaneous, i.e. the source knows (a) whether or not a packet is lost and (b) identity of the receiver with the lost packet instantaneously. This implicitly assumes that ACK/NAKs are never lost. This assumption is not critical as we can incorporate the delay and bandwidth used by ACK/NAKs into the analysis.
4) Packet loss at a receiver i follows the Bernoulli distribution with parameter p_i. In addition, the packet loss at different receivers are uncorrelated. This model is clearly insufficient to describe many real-world scenarios. However, this model is only intended for capturing the essence of wireless broadcast. One can develop a more accurate model, albeit complicate analysis.

A. Broadcast Schemes without Network Coding

Scheme A *(Memoryless receiver)*. In this scenario, a receiver sends a NAK immediately whenever there is a packet loss in the current time slot, regardless whether it has received this packet correctly in some previous time slots (hence memoryless). This situation arises when a receiver received a correct packet, but this packet was lost at some other receivers at some previous time slots. Hence, the source has to retransmit this packet. If this packet is now lost in the current time slot, a memoryless receiver would automatically request a retransmission, even though it has correctly received the packet before. This scheme is clearly suboptimal in terms of bandwidth utilization as it implies that the source has to resend a packet until *all* the receivers receive this packet correctly and *simultaneously*.

Scheme B. In this scenario, a receiver sends a NAK immediately only if there is a packet loss in the current time slot and this packet has not been received correctly in any previous time slot. This scheme is clearly superior to scheme A in terms of bandwidth utilization. Consider the following scenario with one source and two receivers R_1 and R_2. Suppose in the first time slot, a packet is correctly received at R_1, but not at R_2. So, the source has to rebroadcast this packet. In the second time slot, the packet is received correctly at R_2, but not at R_1. Using scheme A, the source has to retransmit the packet the third time because of the memoryless receivers. On the other hand, using scheme B, neither R_1 nor R_2 will send a NAK, and therefore the source can send a different packet, resulting in better bandwidth utilization.

B. Broadcast Schemes with Network Coding

Scheme C *(Time-based retransmission)*. In this scheme, the receiver's protocol is similar to that of the receiver in scheme B in which it sends the NAK immediately if it does not receive a packet correctly. However, the source does not retransmit the lost packet immediately when it receives a NAK. Instead, the source maintains a list of lost packets and the corresponding receivers for which their packets are lost. The retransmission phase starts at a fixed interval of time in terms of number of time slots N, e.g. $N = 100$. During the retransmission phase, the source forms a new packet by XORing a maximum set of the lost packets from different receivers before retransmitting this combined packet to all the receivers. The *combined* packets may be lost during the retransmission, and these packets will be retransmitted until all the receivers receive this packet. The source keeps sending out the *combined* packets until no more lost packets on the list, it then resumes the transmission of a different set of packet. Even though a receiver successfully receives the *combined* packets, it must be able to recover the lost packets, and it does so by XORing this combined packets with appropriate set of previously successful packets. The information on choosing this appropriate set of packets are included in the packets sent by the source. For example, Fig. 1 shows a pattern of lost packets (denoted by the crosses) for two receivers R_1 and R_2. The combined packets are $1 \oplus 3$, $4 \oplus 5$, 7, 9. Note that

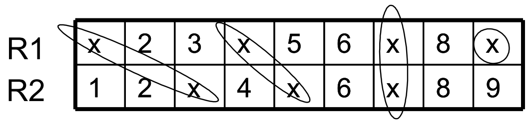

Fig. 1. *Combined packets for time-based retransmission:* $1 \oplus 3$, $4 \oplus 5$, *7, 9;* $M = 9$

if packet $1 \oplus 3$ is not received correctly at any receiver, this packet is retransmitted until all the receiver receives this packet correctly, but may not be simultaneously. Receiver R_1 recovers packet 1 as $3 \oplus (1 \oplus 3)$. Similarly, receiver R_2 recovers packet 3 as $1 \oplus (1 \oplus 3)$. When there is the same loss at both receivers R_1 and R_2, the encoding process is not needed and the source just has to retransmit that packet alone. Note that the source has to include some bits to indicate to a receiver which set of packets it should use for XORing. Assuming that all the retransmissions are correctly received at all the receivers at the first attempt, then clearly the number of retransmissions for this scheme is only 4 while it is 6 for scheme B.

Scheme D *(Improved time-based retransmission).* Scheme C is suboptimal because the source has to retransmit the same combined packet even though some receivers may receive it. An improved scheme is to have the source dynamically changes the combined packets based on what the receivers have received. For example, Fig. 2 shows a same pattern of lost packets as in the previous scenario. Now, suppose the packet $1 \oplus 3$ is lost at receiver R_2, but is received correctly at receiver R_1. In this case, instead of retransmitting packet $1 \oplus 3$, the source can transmit packet $3 \oplus 4$. Clearly, on average, the number of transmissions can be further reduced using this scheme.

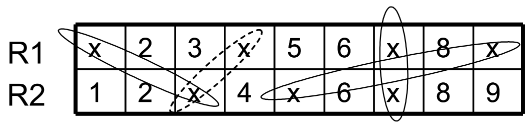

Fig. 2. *Combined packets for improved time-based retransmission:* $1 \oplus 3$, $3 \oplus 4$, $5 \oplus 9$, *6;* $M = 9$.

Remarks: Note that a larger buffer size N results in better bandwidth efficiency, however, it may incur unnecessary long delay for a packet. This may be acceptable for file transfer, but not multimedia applications. When $N = 1$, the network coding scheme reduces to the scheme B. In the following section, we derive a few theoretical results on transmission bandwidth for different schemes.

IV. Transmission Bandwidth Analysis

We define the transmission bandwidth as the average number of transmissions required to successfully transmit a packet to all the receivers. Let η_A, η_B, η_C, and η_D denote the transmission bandwidth using schemes A, B, C, and D, respectively. Let M denote the number of receivers, and p_i's denote the packet loss probability of receiver i. We first discuss the non network coding schemes A and B.

A. Non Network Coding Schemes A and B

We begin with a special case where there are only two receivers with the packet loss probabilities of p_1 and p_2. We have the following results:

Proposition 4.1: The transmission bandwidth of scheme A with two receivers is:

$$\eta_A = \frac{1}{(1-p_1)(1-p_2)}, \quad (1)$$

and using scheme B is:

$$\eta_B = \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2} \quad (2)$$

Proof: For scheme A, the proof is simple. As described in Section III, the source has to retransmit the packets until both the receivers receives the correct packets simultaneously. Since the packet loss is independent and uncorrelated between the receivers (Bernoulli distribution), the number of transmission attempts before both receivers correctly receives the data follows the geometric distribution with the parameter $(1-p_1)(1-p_2)$. Therefore, the average number of retransmissions per a successful event is $1/(1-p_1)(1-p_2)$. Thus, the total number of transmission for N successful events is $N/(1-p_1)(1-p_2)$.

For scheme B, let X_1, X_2 be the random variables denoting the numbers of attempts to successfully deliver a packet to R_1 and R_2, respectively. Then the number of retransmissions needed to successfully deliver a packet to both receivers is the random variable $Y = max\{X_1, X_2\}$. We have:

$$P[Y \le k] = P[X_1 \le k]P[X_2 \le k] = (1-p_1^k)(1-p_2^k) \quad (3)$$

Therefore,

$$P[Y = k] = (1-p_1^k)(1-p_2^k) - (1-p_1^{k-1})(1-p_2^{k-1}). \quad (4)$$

Next,

$$\begin{aligned} E[Y] &= \sum_{k=0}^{\infty} k((1-p_1^k)(1-p_2^k) - (1-p_1^{k-1})(1-p_2^{k-1})) \\ &= \sum_{k=0}^{\infty} k(p_1^{k-1} - p_1^k) + \sum_{k=0}^{\infty} k(p_2^{k-1} - p_2^k) \\ &+ \sum_{k=0}^{\infty} k(p_1^k p_2^k - p_1^{k-1} p_2^{k-1}) \\ &= \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2} \quad (5) \end{aligned}$$

■

We can generalize the result to networks with more than 2 receivers. We have the following theorem:

Theorem 4.1: The transmission bandwidth of scheme A with M receivers is:

$$\eta_A = \frac{1}{\prod_{i=1}^{M}(1-p_i)} \tag{6}$$

and using scheme B is:

$$\eta_B = \sum_{i_1,i_2,...,i_M} \frac{(-1)^{i_1+i_2+...i_M-1}}{1-p_1^{i_1}p_2^{i_2}...p_M^{i_M}} \tag{7}$$

where $i_1, i_2, ..., i_M \in \{0, 1\}$

Proof: The proof is provided in [9]. ■

B. Network Coding Schemes C and D

Unlike the schemes A and B, scheme C has one additional parameter, namely, the size of the buffer used to maintain a list of receivers and their corresponding lost packets. When a small buffer is used, there may not be sufficiently many lost packets for generating the combined packets which can reduce the bandwidth efficiency. On the other hand, when a large buffer is used, the bandwidth efficiency improves at the expense of packet delay. This approach is acceptable for file transfer applications. We now provide an asymptotic result when the buffer size N and the number of packets to be sent T are sufficiently large. Since it is not beneficial to have $N > T$, we assume $T = N$ and N is sufficiently large. We have the following results for two receivers.

Proposition 4.2: The transmission bandwidth of scheme C with two receivers where $p_1 \leq p_2$ *and N is sufficiently large is:*

$$\eta_C = 1 + \frac{p_1}{1-p_2} + \frac{p_2}{1-p_1} - \frac{p_1}{1-p_1p_2} \tag{8}$$

Proof: The key to our proof is the following observation. The transmission bandwidth depends on how many pairs of lost packets one can find to generate combined packets. When N, the number of packets to be sent is sufficiently large, the probability that the number of lost packets at the receiver R_1 is smaller than that of receiver R_2, is arbitrarily close to 1. Furthermore, the average numbers of lost packets for R_1 and R_2 are Np_1 and Np_2, respectively. This implies that on average the number of packets one can pair up is Np_1 since $Np_1 \leq Np_2$. This means there are $Np_2 - Np_1$ lost packets from R_2 that need to be retransmitted alone. Therefore, the total number of transmissions required to successfully deliver all N packets to two receivers is simply

$$n = N + Np_1E[X_1] + N(p_2 - p_1)E[X_2], \tag{9}$$

Where X_1 and X_2 are the random variables denoting the numbers of transmission attempts before a successful transmission for the combined and non-combined packets. Now, $E[X_2] = \frac{1}{1-p_2}$ since X_2 follows the geometric distribution. From Proposition 4.1, we have

$$E[X_1] = \frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1p_2}$$

Replacing $E[X_1]$ and $E[X_2]$ in Equation (9) and divide n by N, we obtain the result of Proposition 4.2. ■

We can generalize the result to M receivers.

Theorem 4.2: The transmission bandwidth of scheme C with M receivers and sufficiently large N is:

$$\eta_C = 1 + p_1\varphi_M + \sum_{j=1}^{M-1}(p_{j+1} - p_j)\varphi_{M-j} \tag{10}$$

where,

$$\varphi_M = \sum_{i_1,i_2,...,i_M} \frac{(-1)^{i_1+i_2+...+i_M-1}}{1-p_1^{i_1}p_2^{i_2}...p_M^{i_M}} \tag{11}$$

where $i_1, i_2, ...i_M \in \{0, 1\}$ *and* $p_1 \leq p_2 \leq ... \leq p_m$.

Proof: Since $p_1 \leq p_2 \leq ... \leq p_M$, after a sufficiently large number of transmissions N, the number of packet losses at receivers $R_1, R_2, ..., R_M$ are $Np_1, Np_2, ..., Np_m$ and $Np_1 \leq Np_2 \leq ... \leq Np_M$. We can conceptually count the number of combinations for XORing the lost packets and transmit these packets in different rounds. In particular, in round 1, there are Np_1 lost packets of R_1 that can be combined with the lost packets of $R_2, R_3, ..., R_M$. After these combinations, the numbers of lost packets remain for R_1, R_2, R_3, ... R_M are 0, $N(p_2 - p_1)$, $N(p_3 - p_1)$, ... $N(p_M - p_1)$, respectively. Next in round 2, the remaining $N(p_2 - p_1)$ lost packets at R_2 are combined with the remaining lost packets at R_3, R_4, ... R_M. Thus, the remaining lost packets for receivers R_1 to R_M are now 0, 0, $N(p_3 - p_2)$, ... $N(p_M - p_{M-1})$. The same reasoning applies until there are no more lost packets. Therefore, the average number of transmissions required to successfully delivers all N packets to all the receivers equals to

$$\begin{aligned} n &= N + Np_1\phi_1 + N(p_2 - p_1)\phi_2 + N(p_3 - p_2)\phi_3 \\ &+ ... + N(p_M - p_{M-1})\phi_M, \end{aligned} \tag{12}$$

where ϕ_i denotes the average number of transmissions required to successfully transmit a combined packet in round i. We note that in the first round there are M receivers with lost packets, in the second round there are $M-1$ receivers with lost packets, and the number of receivers with lost packets in each round decreases until there is only a single receiver R_M. Now, using Theorem 4.1, the average number of transmission attempts in order for all K receivers to correctly receive a packet is:

$$\begin{aligned} \varphi_K &= \left(\frac{1}{1-p_1} + ... + \frac{1}{1-p_K}\right) \\ &+ (-1)^1\left(\frac{1}{1-p_1p_2} + \frac{1}{1-p_1p_3} ... + \frac{1}{1-p_{K-1}p_K}\right) \\ &+ ... + (-1)^{K-1}\left(\frac{1}{1-p_1p_2...p_K}\right) \end{aligned} \tag{13}$$

Or,

$$\varphi_K = \sum_{i_1,i_2,...i_M} \frac{(-1)^{i_1+i_2+...i_M-1}}{1-p_1^{i_1}p_2^{i_2}...p_K^{i_K}} \tag{14}$$

where $i_1, i_2, ...i_M \in \{0, 1\}$ and $p_1 \leq p_2 \leq ... \leq p_m$. Therefore, we can set $\phi_i = \varphi_{M+1-i}$. Divide n by N, the proof follows directly.
When $p_1 = p_2 = ... = p_M = p$, η_C becomes:

$$\eta_C = 1 + p\eta_B \tag{15}$$

■

Theorem 4.3: The transmission bandwidth of scheme D with M receivers and sufficiently large N is:

$$\eta_D = \frac{1}{1 - max\{p_1, p_2, ..., p_M\}} \tag{16}$$

Proof: We begin with the case of two receivers. Without loss of generality, we assume that $p_1 \leq p_2$. As discussed in Section III, the combined packets in scheme D are dynamically formed based on the feedback from the receivers. If a combined packet is correclty received at some receivers, but not at others, a new combined packet is generated to ensure that the receivers with the correct packet will be able to obtain the new data using the new combined packet. This implies that after a long run, the number of losses will be dominated by the number of losses at the receiver with the largest error probability (R_2). Therefore, the total number of transmissions to successfully deliver N packets to two receivers equals to the number of transmissions to successfully deliver N packets to R_2 alone, i.e. $\frac{N}{1-p_2}$ or $\frac{N}{1-max\{p_1,p_2\}}$. Without much difficulty, we can generalize this result to the network with M receivers:

$$n = \frac{N}{1 - max\{p_1, p_2, ..., p_M\}} \tag{17}$$

Therefore, the transmission bandwidth is:

$$\eta_D = \frac{n}{N} = \frac{1}{1 - max\{p_1, p_2, ..., p_M\}} \tag{18}$$

■

Theorem 4.4: The transmission bandwidth of scheme D with M receivers and buffer size N is:

$$\eta_D^N = \frac{\sum_{k=N}^{\infty} k(\prod_{j=1}^{M} \sum_{i=0}^{k-N} P(X_j, N, i) - \prod_{j=1}^{M} \sum_{i=0}^{k-N-1} P(X_j, N, i))}{N} \tag{19}$$

where $P(X_j, N, i) = {p_j}^i (1-p)^N \sum_{l=1}^{i} \binom{N}{l}\binom{i-1}{l-1}$ if $i \leq N$, and $P(X_j, N, i) = {p_j}^i (1-p)^N \sum_{l=1}^{N} \binom{N}{l}\binom{i-1}{l-1}$ if $i > N$.

Proof: Due to limited space, the proof can be found in [9]. ■

C. Network Coding Gain

In previous section, we analyze the transmission bandwidth of different schemes. Here we show an example of coding gains of schemes C and D over scheme B for two receivers. Coding gains are defined as:

$$G_C = \frac{\eta_B}{\eta_C} = \frac{\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2}}{1 + \frac{p_2}{1-p_1} + \frac{p_1}{1-p_2} - \frac{p_1}{1-p_1 p_2}} \tag{20}$$

$$G_D = \frac{\eta_B}{\eta_D} = \frac{\frac{1}{1-p_1} + \frac{1}{1-p_2} - \frac{1}{1-p_1 p_2}}{\frac{1}{1-max\{p_1,p_2\}}} \tag{21}$$

With $p_1 = p_2 = p$, we have

$$G_C = \frac{1 + 2p}{1 + p + p^2} \tag{22}$$

$$G_D = \frac{1 + 2p}{1 + p} \tag{23}$$

We show a few more results on coding gain in the next section.

V. Simulation Results

In this section, we present the simulation results on the transmission bandwidth and coding gain of schemes C and D over scheme B. We choose not to discuss scheme A since this scheme has very poor bandwidth utilization. Fig. 3 shows the simulation results and the theoretical results on the bandwidth efficiency for the scenario consisting of two receivers R_1 and R_2. The packet loss probability of R_1 varies as shown on the x-axis while that of R_2 remains at 10%. As seen, the number of transmissions per packet in scheme D is smallest while that of scheme B is largest. Scheme D is more efficient than scheme C. We note that the hardware implementation of scheme D might be little more complex than that of scheme C due to its dynamic selection of the retransmission packets. The coding gains of schemes C and D are shown in Fig. 4.

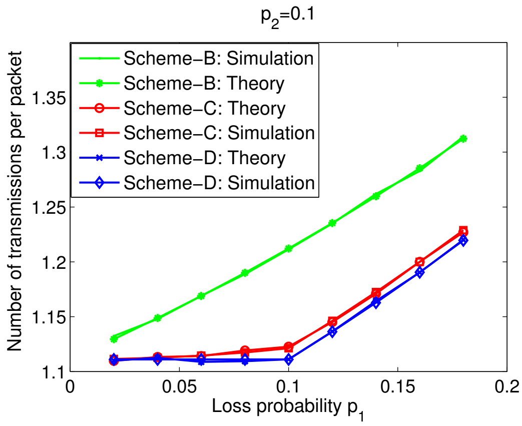

Fig. 3. *Transmission bandwidth versus packet loss probability.*

The gain is largest when both loss probabilities of R_1 and R_2 are equal to each other. This is intuitively plausible as in this special case, the maximum number of lost packet pairs is achieved. When the loss probability of one receiver is much larger than other, network coding is less useful since it has to spend the bandwidth on retransmitting the non-combined packets of the receiver with larger loss probability. For this scenario, the gain is not much, only from 2% to 9%. We note that the coding gain depends on (a) the loss probabilities and (b) the number of receivers. To quantitatively show the coding gain as a function of the number of the receivers, Fig. 5 shows the average number of transmissions required to successfully deliver a packet to all the receivers for schemes B, C and D. In this scenario, the loss probabilities of all the receivers are set to 0.1. The network coding schemes C and

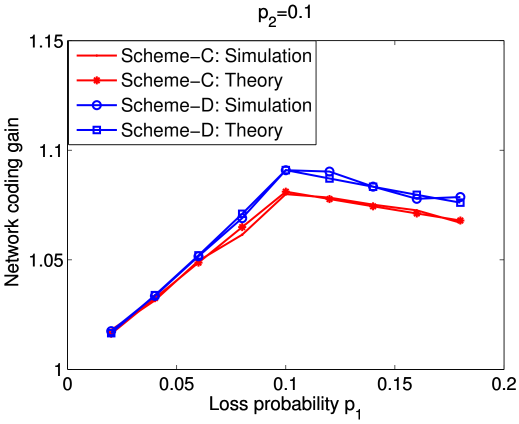

Fig. 4. *Network coding gain versus the packet loss probability.*

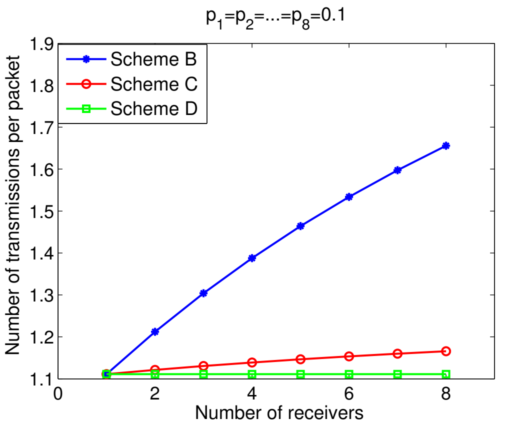

Fig. 5. *Transmission bandwidth versus the number of receivers.*

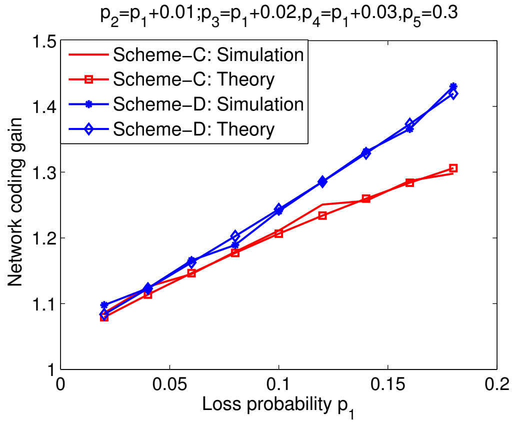

Fig. 6. *Network coding gain versus the packet loss probability.*

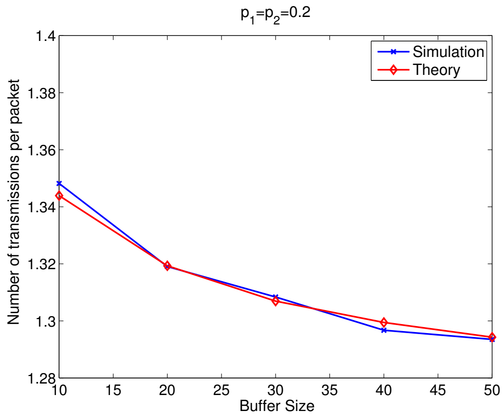

Fig. 7. *Transmisison bandwidth versus the buffer size for scheme D.*

D significantly outperform scheme B when there is a large number of receivers. One interesting observation is that as the number of receivers increases, the transmission bandwidth for scheme C increases very slightly and is unchanged for scheme D. This implies that scheme C and D scale very well with the number of receivers. Fig. 6 shows the theoretical and simulated coding gains for five receivers with different loss probabilities for schemes C and D. As seen, the simulation results verify our theoretical predictions. Finally, Fig. 7 shows the transmission bandwidth as a function of the buffer size for scheme D. As expected, as the buffer size increases, the opportunities for combining lost packets increases, thus resultls in smaller retransmission overhead.

VI. Conclusion

In this paper we propose some network coding techniques to increase the bandwidth efficiency of reliable broadcast in a wireless network. Our proposed schemes combine different lost packets from different receivers in such a way that multiple receivers are able to recover their lost packets with one transmission by the source. The advantages of the proposed schemes over the traditional wireless broadcast are shown through simulations and theoretical analysis. Specifically, we provide a few results on the transmission bandwidth of the proposed scheme under different channel conditions.

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