Concatenated Network Coding for Large-Scale Multi-Hop Wireless Networks

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Abstract-We present "concatenated" network coding technique that incorporates multi-level coding across a network of relays in large-scale, multi-hop, noisy wireless networks. A key feature of the proposed approach is that more powerful code can be constructed, hence more errors can be corrected, as the information travels through the network. The decomposition of large-scale network into several small-scale networks and concatenation of small-scale network codes along the route path simplifies both encoding (forwarding) and decoding processes. This is particularly important, because the nodes need to make block encoding based on local information only. This framework is inherently self scalable, in that each new node and each additional hop contribute reliability enhancement. We analyze the probability of decoding error and the normalized throughput as a function of network configuration and number of decoding iterations.

Index Terms- Concatenated network coding, cooperative relaying, iterative decoding, multi-user wireless networks.

I. Introduction

The concept of network coding has evolved recently as an interesting extension of the more traditional routing paradigm. The basic idea of network coding is to allow coding capability at relaying (intermediate) nodes. Instead of simply forwarding (repeating) the stored data, relay nodes may combine several input packets, encode them, and forward it to the next node. It exploits the broadcast property offered by the wireless medium that transmitted signals can be received and processed by any node in the neighborhood of a transmitter. The motivation is that there are codes more powerful than repetition coding. This allows a much larger degree of freedom in the way packets are combined and forwarded.

The pioneering work in [1] has thereafter inspired considerable research efforts in computer networking and communication communities. Though most of these studies have focused on wired networks with error-free transmission, there were some initial works investigating network coding in wireless networks [2],[3]. Then, it was discussed how to gain cooperative diversity through network coding in wireless networks to combat channel fading in [4]-[7]. The concept of cooperative diversity [8] was initially applied (without network coding) for the relay channel. Then, the network coding gains of various cooperative diversity protocols have been examined for a single-source and single-destination scenario in [9]. Most of these works, however, are limited to small scale networks with one or two hops.

Fig. 1 shows a simple network illustrating the basic idea of the proposed network coding technique in multi-access relay channel. In a wireless network consisting of K sources, R relays, and one destination, suppose that each of K sources has its own independent messages $m_1, m_2, ..., m_K$ to be sent to the destination N_A. Examples of this scenario are uplink cellular networks, sensor networks, or downloading high bandwidth applications (e.g. video) or bulk file from different servers. Due to the broadcast nature of the wireless medium, assume each relay node can also receive signals $m_1, m_2, ..., m_K$, possibly with some errors. Then, each relay node generates a parity-check symbol p_i, $i = 1, 2, ..., R$, based on what it receives from K sources and transmits it to the destination. This simpifies the routing problem, because we do not have to worry about which message (packet) to forward to the other node: we combine *all* received messages. Then, the destination may construct a noisy $(K+R, K)$ codeword of rate $r = K/(K+R)$ consisting of K message bits and R parity-check bits. The minimum distance of such codes can be as large as R+1 (Singleton bound), providing a diversity gain R+1 for K sources with the help of R relays.

In this paper, we will extend this idea to a *multi-hop* scenario in large scale wireless networks and present a "concatenated" network coding technique. Concatenated coding, devised by Forney [10], is an effective method for constructing *long* powerful codes from short component codes with reduced decoding complexity. The proposed concatenated network coding incorporates multi-level coding of multiple sources across a network of relays over multi-hops, where parity bits are generated at the relays based on the noisy codewords of previous nodes and transmitted to the next nodes until the information reaches its final destination. A *key feature* of the proposed approach is that more powerful code can be constructed, hence more errors can be corrected, as the information travels through the network. The decomposition of large-scale network into several small-scale networks and concatenation of small-scale network codes along the route path simplifies both encoding (forwarding) and decoding processes. This is particularly important, because the nodes need to make block encoding based on local information only. This framework is inherently self scalable, in that each new node and each additional hop contribute reliability enhancement. We will derive the probability of decoding error at the destination and the network throughput as a function of network configuration and number of decoding iterations.

II. System Description

Fig. 2 illustrates the proposed concatenated network coding technique in cooperative relay networks. We assume that nodes are clustered such that there are K_1 source nodes, R_1 relay nodes, and one cluster head in each cluster (small square), and that there are K_2 clusters communicating with node N_B (base station or another cluster head) through R_2 relay nodes. It is assumed that nodes are clustered using some node clustering techniques, such as those described in [11],[12].

Each cluster head N_A constructs a (K_1+R_1, K_1) "outer code" C_1 by combining all message bits and parity-check bits generated by the relay nodes in its cluster. If the cluster head decodes correctly, then it broadcasts its (outer) codeword to R_2 relays and node N_B. If a decoding failure occurs at least at one cluster head, then corresponding vector will not be forwarded to N_B and decoding failure is declared on K_1xK_2 block at node N_B. Once all cluster heads decode correctly, then each of R_2 relays stores all noisy outer codewords sent from K_2 cluster heads in a K_2x(K_1+R_1) array. Then, each column of the array is used to generate the parity-check bits for the "inner code" C_2 at each relay node which are then transmitted to the destination N_B. This allows the destination N_B to construct a two-dimensional concatenated code C_1xC_2, shown in Fig. 3. Its minimum distance is d_1d_2, where d_1 and d_2 are minimum distance of code C_1 and code C_2, respectively [13]. Since $d_1d_2 \le (R_1+1)(R_2+1)$, the proposed approach can provide a maximum diversity gain of $(R_1+1)(R_2+1)$ for K_1K_2 sources with R_1+R_2 relays. Since the minimum distance is increased from d_1 at node N_A to d_1d_2 at node N_B, more errors can be corrected as the information travels through the network. This concept can be extended to more than two-level concatenation, C_1xC_2x ... xC_n, as illustrated in Fig. 4.

In decoding the message at the destination N_B, each column vector of length K_2+R_2 is first decoded based on the inner code C_2, capable of correcting t_2 $(\le R_2/2)$ errors. If a decoding error occurs, which happens if there are more than t_2+1 errors in a column vector, then the corresponding column vector is erased. Here we assume that the probability of undetected error is much smaller than that of decoding failure [13]. After K_1+R_1 column (inner code) decodings are completed, the parity bits of C_2 (R_2 rows) are removed, leaving an array of size K_2x(K_1+R_1). Then, each row of the array is decoded based on the outer code C_1 using an erasure decoding algorithm. At the completion of outer decoding, the K_1K_2 decoded information bits are delivered to N_B. If needed, the intermediate cluster head N_A can also decode and retrieve the information within its cluster, namely $m_1, m_2, ...m_{K_1}$.

We will assume BPSK modulation at both sources and relays with symbol energies $E_{b,s}$ and $E_{b,r}$, respectively. The total transmit energy E_b in sending one *information* bit from source to destination in a cluster is then given by

$$\begin{aligned} E_b &= (KE_{b,s}+RE_{b,r})/K \\ &= E_{b,s}+(R/K)E_{b,r}. \end{aligned} \tag{1}$$

Channels are divided in time or frequency, and thus have no interference.

III. Performance Analysis

We will analyze the performance of network coded cooperative relaying within a single cluster and then move on to concatenated network coded system. We will consider hard decision decoding for simplicity of processing at the destination or cluster head, which is essential in typical sensor networks due to severe computing resource constraints.

Let p, q, and p_r be the bit error probability for the source-to-destination, source-to-relay, and relay-to-destination links, respectively, in a cluster. Let $\hat{m}_i$ and $\hat{\hat{m}}_i$ be the estimation of m_i at the relay and the destination, respectively. Also, let $\hat{p}_j$ be the parity bit generated at the j-th relay based on $\{\hat{m}_i\}$, and $\hat{\hat{p}}_j$ be its estimation at the destination. Then, the received vector at the destination N_A in Fig. 1 is $(\hat{\hat{m}}_1, \hat{\hat{m}}_2, ..., \hat{\hat{m}}_K, \hat{\hat{p}}_1, ..., \hat{\hat{p}}_R)$.

For BPSK modulation in Rayleigh flat fading channel, it is known that [14]

$$p = P(\hat{\hat{m}}_i \ne m_i) = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{SD}}{1+\bar{\gamma}_{SD}}}\right) \tag{2}$$

$$q = P(\hat{m}_i \ne m_i) = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{SR}}{1+\bar{\gamma}_{SR}}}\right) \tag{3}$$

$$p_r = P(\hat{\hat{p}}_i \ne \hat{p}_i) = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{RD}}{1+\bar{\gamma}_{RD}}}\right) \tag{4}$$

where

$$\bar{\gamma}_{SD} = d_{SD}^{-m}E_{b,s}/N_0 \tag{5}$$

$$\bar{\gamma}_{SR} = d_{SR}^{-m}E_{b,s}/N_0 \tag{6}$$

$$\bar{\gamma}_{RD} = d_{RD}^{-m}E_{b,r}/N_0 \tag{7}$$

are average received signal-to-noise ratios, where m is the path loss exponent, $r = K/(K+R)$ is the code rate, and d_{SD}, d_{SR}, and d_{RD} are normalized distances of source-to-destination, source-to-relay, and relay-to-destination links, respectively. Throughout this paper we will assume that $d_{SD} = 1$ (normalized to one), $d_{SR} = \alpha$, and $d_{RD} = 1-\alpha$, where $0 \le \alpha \le 1$.

The probability of $\hat{\hat{p}}_j$ not being equal to p_j, $\tilde{p}$, is given by

$$\begin{aligned} \tilde{p} &= P(\hat{\hat{p}}_j \ne p_j) \\ &= P(\hat{\hat{p}}_j \ne p_j | \hat{p}_j = p_j)P(\hat{p}_j = p_j) \\ &\quad + P(\hat{\hat{p}}_j \ne p_j | \hat{p}_j \ne p_j)P(\hat{p}_j \ne p_j) \\ &= (1-2p_r)P(\hat{p}_j \ne p_j) + p_r \end{aligned} \tag{8}$$

where

$$P(\hat{p}_j \ne p_j) = \frac{2^{R-1}}{2^R-1}\left[1-(1-q)^K\right] \tag{9}$$

is the probability that the parity bit generated at a relay is incorrect due to an incorrect reception of message bits, i.e. $\hat{m}_i \neq m_i$, for some $i \in \{1, 2, ..., K\}$. In deriving (9), we assumed that the parity bit error patterns are equi-probable if the message vector received at a relay is in error.

A. Two-Level Concatenation

The probability of decoding failure at node N_A (cluster head) in Fig. 2 is given by

$$P_A = \sum_{i=0}^{K_1} \sum_{j=(t_1+1-i)_+}^{R_1} \binom{K_1}{i} p^i (1-p)^{K_1-i} \binom{R_1}{j} \tilde{p}^j (1-\tilde{p})^{R_1-j} \quad (10)$$

where $\tilde{p}$ is given by (8) with R and K in (9) replaced by R_1 and K_1, respectively, and t_1 is the error correction capability of (K_1+R_1, K_1) code (C_1). Since there are K_2 such clusters, the probability that all K_2 cluster heads decode correctly is $(1-P_A)^{K_2}$.

At the next level, the probability of decoding failure on a column vector in Fig. 3, hence the probability of bit erasure in a row vector, is given by

$$P_2 = \sum_{i=0}^{K_2} \sum_{j=(t_2+1-i)_+}^{R_2} \binom{K_2}{i} p^i (1-p)^{K_2 1-i} \binom{R_2}{j} \tilde{p}^j (1-\tilde{p})^{R_2-j} \quad (11)$$

where $\tilde{p}$ is given by (8) with R and K in (9) replaced by R_2 and K_2, respectively, and t_2 is the error correction capability of (K_2+R_2, K_2) code (C_2). If we let d_1 be the minimum distance of (K_1+R_1, K_1) code (C_1), then the probability of decoding failure on a row vector (K_1 information bits) at node N_B is

$$P_1 = \sum_{i=d_1}^{K_1+R_1} \binom{K_1+R_1}{i} P_2^i (1-P_2)^{K_1+R_1-i}. \quad (12)$$

because up to $d_1 - 1$ erasures can be corrected. Hence, the block decoding error probability on K_1xK_2 block is given by

$$P_{E,B} = 1 - (1-P_1)^{K_2} \cdot (1-P_A)^{K_2}. \quad (13)$$

For systems with no network coding ($R_1 = R_2 = 0$)

$$P_{E,B} = 1 - (1-p)^{2K_1K_2}. \quad (14)$$

The normalized throughput, W_B, is given by

$$W_B = \frac{K_1K_2(1-P_{E,B})}{(K_1+R_1)(2K_2+R_2)} \quad (15)$$

where the denominator represents the number of channel uses per K_1xK_2 information bits.

Iterative Decoding: Concatenated codes can be decoded in iterative manner to further improve error performance with reduced decoding complexity. In iterative decoding, component codes are decoded one at a time in a sequence of decoding stages. The decoded information at one stage is passed to the next stage for decoding the next component code.

If we let $P_2(m)$ be the probability of decoding failure on a column vector after m iterations, then the probability of decoding failure on a row vector after m iterations is given by

$$P_1(m) = \sum_{i=d_1}^{K_1+R_1} \binom{K_1+R_1}{i} P_2(m)^i (1-P_2(m))^{K_1+R_1-i}. \quad (16)$$

Hence, the block decoding error probability at node N_B after m iterations is given by

$$P_{E,B}(m) = 1 - (1-P_1(m))^{K_2} \cdot (1-P_A)^{K_2}. \quad (17)$$

The probability of decoding failure on a column vector after $m+1$ iterations is then given by

$$P_2(m+1) = \sum_{i=d_2}^{K_2+R_2} \binom{K_2+R_2}{i} P_1(m)^i (1-P_1(m))^{K_2+R_2-i}. \quad (18)$$

Decoding iterations continue until a certain stopping criterion is satisfied.

B. Three-Level Concatenation

For a three-level concatenation code C_1xC_2xC_3, illustrated in Fig. 5 that corresponds to the network configuration shown in Fig. 4, the probability of decoding failure of C_3 codeword is given by

$$P_3 = \sum_{i=d_3}^{K_3+R_3} \binom{K_3+R_3}{i} P_1^i (1-P_1)^{K_3+R_3-i} \quad (19)$$

where d_3 is the minimum distance of (K_3+R_3, K_3) code (C_3). Hence, the block decoding error probability on K_1xK_2xK_3 block at node N_C is given by

$$P_{E,C} = 1 - (1-P_3)^{K_1K_2} \cdot (1-P_{E,B})^{K_3}. \quad (20)$$

The normalized throughput for three-level concatenation, W_C, is given by

$$W_C = \frac{K_1K_2K_3(1-P_{E,C})}{(K_1+R_1)(2K_2+R_2)(2K_3+R_3)}. \quad (21)$$

This process can be generalized to an arbitrary multi-level concatenation code C_1xC_2x ... xC_n for an arbitrary multi-hop network.

IV. Numerical Results and Discussion

Fig. 6 shows the block decoding error probability versus E_b/N_0 with two-level concatenated network coded relaying system. Parity bits are generated at the relays using binary BCH code to construct a (K_1+R_1, K_1)x(K_2+R_2, K_12) concatenated code at the destination. We find that the coding gain can be significantly high, providing up to 22 dB gain at block error probability of 10^{-2}. This is because the proposed concatenated network coding can correct more errors, as the information travels through the network. We also find that the coding gain depends on the network topology: network topology of $R_1 = 20$ and $R_2 = 5$ provides additional 8.5dB coding gain as compared to that of $R_1 = 5$ and $R_2 = 20$.

Fig. 7 shows the normalized throughput W versus E_b/N_0 in two-level concatenated network coded relaying system. We find that there exists an optimal network topology R_1xR_2 that maximizes the normalized throughput, and the optimal number of relays decreases with increasing SNR.

Fig. 8 shows the block decoding error probability $P_E(m)$ versus E_b/N_0 for different number of iterations in two-level concatenated network coded relaying system. We find that the error performance significantly improves from 1 iteration to 2 iterations, and almost saturates after 2 iterations. There is no improvement beyond 3 iterations.

V. Conclusion

We proposed "concatenated" network coded cooperative relaying technique that incorporates multi-level coding across a network of relays in multi-source, multi-hop wireless networks. A key feature of this approach is that more powerful code can be constructed, hence more errors can be corrected, as the information travels through the network. This framework is inherently self scalable, in that each new node and each additional hop contribute reliability enhancement. It can provide a large diversity gain for a large set of users with limited number of relays and simple decoding complexity at the destination. We analyzed the probability of decoding error and the normalized throughput, and investigated the coding gain (energy saving) that arises from the proposed network coding in multi-hop wireless networks.

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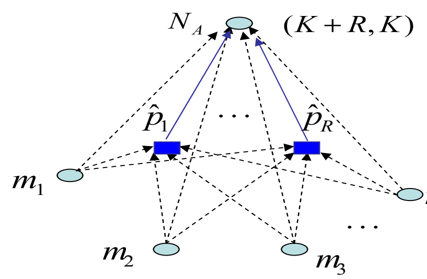

Fig. 1. Network coding in a cluster of K sources, R relays, and one destination.

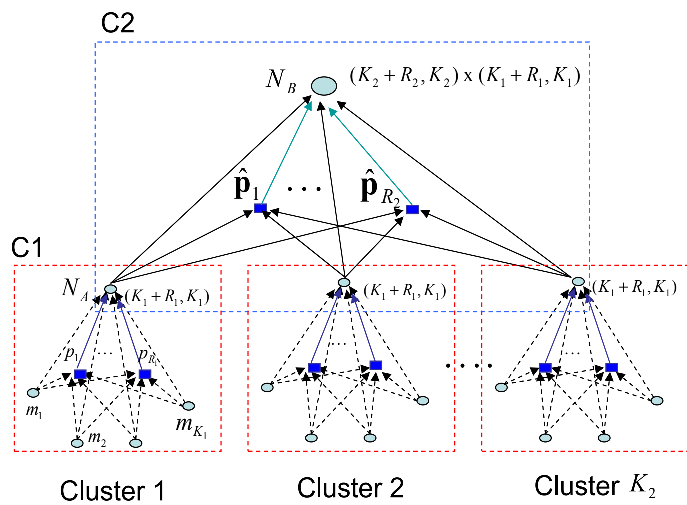

Fig. 2. Concatenated network coding of K_2 clusters with R_2 relays where K_1 sources within each cluster is network coded using R_1 relays.

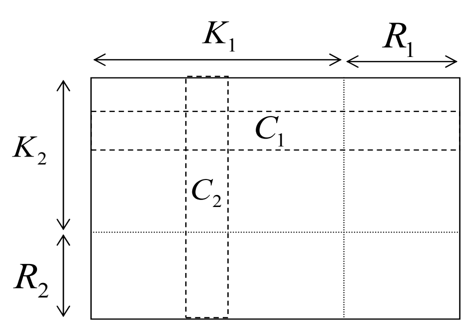

Fig. 3. Two-level (K_1+R_1, K_1)x(K_2+R_2, K_2) concatenated code.

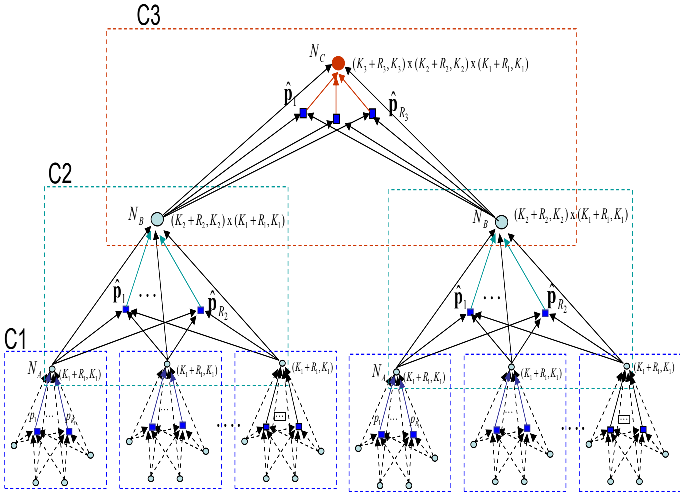

Fig. 4. Concatenated network coding of K_3 clusters with R_3 relays where K_1xK_2 sources within each cluster is network coded using R_1+R_2 relays.

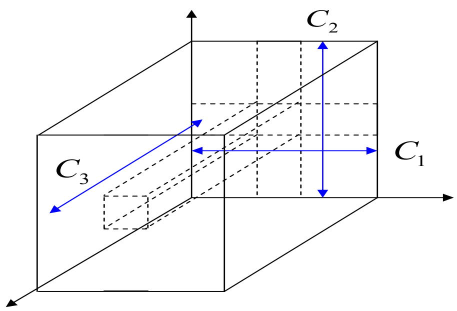

Fig. 5. Three-level $(K_3 + R_3, K_3)$x$(K_2 + R_2, K_2)$x$(K_1 + R_1, K_1)$ concatenated code.

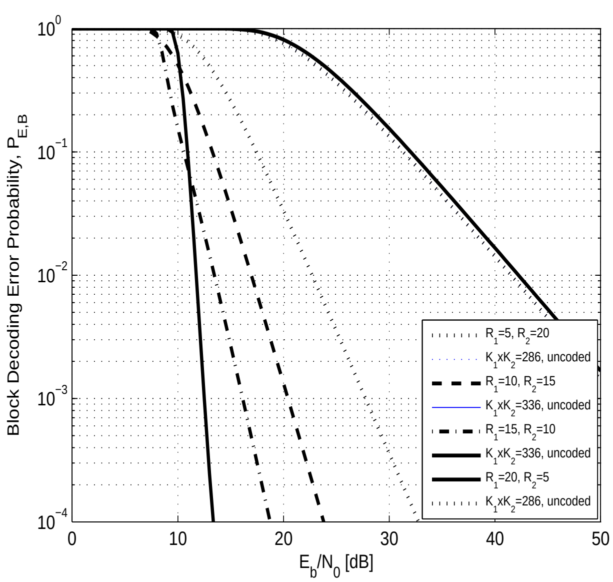

Fig. 6. Block decoding error probability $P_{E,B}$ versus E_b/N_0 with various network topology, two-level concatenated network coding, optimum $E_{b,s}$ and $E_{b,r}$: $\alpha = 0.5$, $m = 4$, $K_1 + R_1 = 31$, $K_2 + R_2 = 31$, binary BCH code.

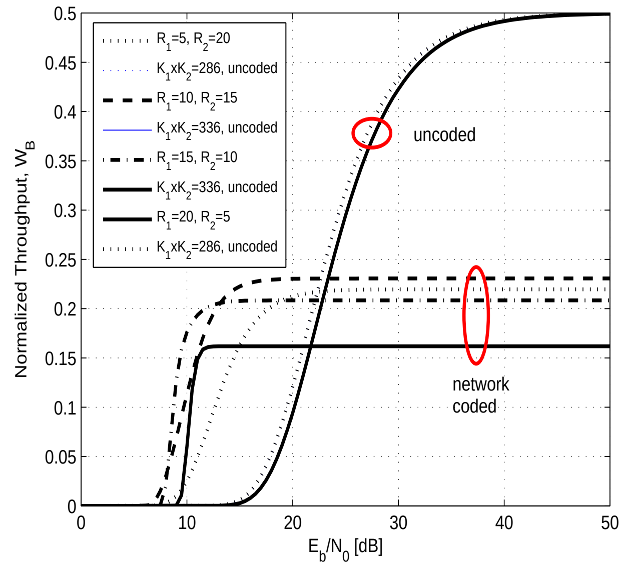

Fig. 7. Normalized throughput W_B versus E_b/N_0 with various network topology, two-level concatenated network coding, optimum $E_{b,s}$ and $E_{b,r}$: $\alpha = 0.5$, $m = 4$, $K_1 + R_1 = 31$, $K_2 + R_2 = 31$, binary BCH code.

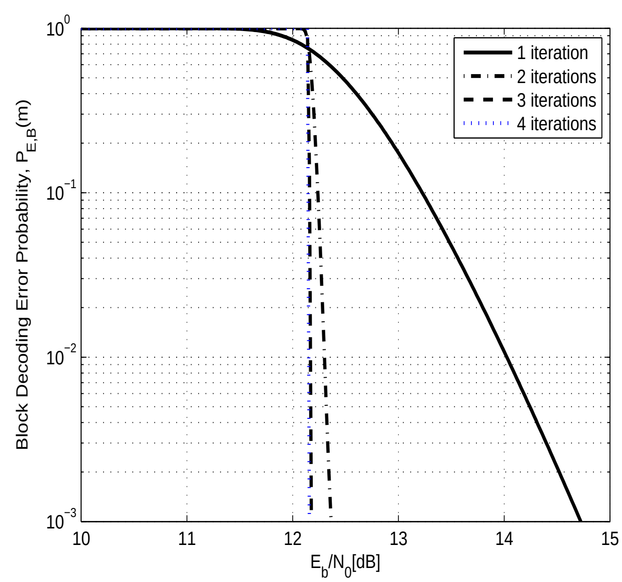

Fig. 8. Block decoding error probability $P_{E,B}(m)$ versus E_b/N_0 for several numbers of iterations, two-level concatenated network coding, $E_{b,s} = 0.5E_b$: $\alpha = 0.5$, $m = 4$, $K_1 + R_1 = 31$, $K_2 + R_2 = 31$, $R_1 = 20$, $R_2 = 5$, binary BCH code.